Temperature calculations for heat transfer in extended sufaces (fins)

Inputs:

Length of the cross-section: $L_f := 100.0mm$

Width & height of cross-section: $B_f := 2.0mm$ $H_f := 5.0mm$

Perimeter: $per := 2 \cdot (H_f + B_f) = 14 \cdot mm$

Cross-section area: $A_c := H_f \cdot B_f = 10 \cdot mm^2$

Heat transfer coefficient: $h_c := 20.0 \frac{W}{m^2 \cdot K}$

Thermal conductivity of fin material: $k_f := 200 \frac{W}{m \cdot K}$

Reference Temperature for HTC: $T_{REF} := 40.0 °C$

Boundary condition base of the fin: $T_B := 200 °C$

Type of boundary condition at tip: Specified Temperature or insulated

Other options can be: convective or infinite length

Boundary condition at tip of the fin: $T_L := 100 °C$

Axial coordinate: $x_f := 0, 2mm .. L_f$

Derived parameters:

Fin constant: $m_f := \sqrt{\frac{h_c \cdot per}{k_f \cdot A_c}} = 11.8 \cdot \frac{1}{m}$

$M := \sqrt{h_c \cdot per \cdot k_f \cdot A_c} = 0.024 \cdot \frac{W}{K}$

Non-dimensional Temperatures: $\theta_B := T_B - T_{REF} = 160 K$

$\theta_L := T_L - T_{REF} = 60 K$

Functions for temperature and heat fluxes:

BC1: Specified convection at tip

$$\theta_1(x) := \theta_B \cdot \frac{\cosh\left[m_f \cdot (L_f - x)\right] + \frac{h_c}{m_f \cdot k_f} \cdot \sinh\left[m_f \cdot (L_f - x)\right]}{\cosh(m_f \cdot L_f) + \frac{h_c}{m_f \cdot k_f} \cdot \sinh(m_f \cdot L_f)}$$

$T_1(x) := \theta_1(x) + T_{REF}$

BC2: Specified temperature at tip

$$\theta_2(x) := \theta_B \cdot \frac{\left(\frac{\theta_L}{\theta_B} + 1\right) \cdot \sinh\left[m_f \cdot \left(L_f - x\right)\right]}{\sinh\left(m_f \cdot L_f\right)} \qquad T_2(x) := \theta_2(x) + T_{REF}$$

BC3: Insulated tip

$$\theta_3(x) := \theta_B \cdot \frac{\cosh\left[m_f \cdot \left(L_f - x\right)\right]}{\cosh\left(m_f \cdot L_f\right)} \qquad T_3(x) := \theta_3(x) + T_{REF}$$

BC4: Infinitely long fin (for $L_f >> B_f$)

$$\theta_4(x) := \theta_B \cdot e^{-m_f \cdot x} \qquad T_4(x) := \theta_4(x) + T_{REF}$$

Heat flux through base of the fin:

BC1: Specified convection

$$q''_{B1}\left(h_c\right) := \sqrt{h_c \cdot per \cdot k_f \cdot A_c} \cdot \theta_B \cdot \frac{\sinh\left(\sqrt{\frac{h_c \cdot per}{k_f \cdot A_c}} \cdot L_f\right) + \sqrt{\frac{h_c \cdot A_c}{k_f \cdot per}} \cdot \cosh\left(\sqrt{\frac{h_c \cdot per}{k_f \cdot A_c}} \cdot L_f\right)}{\cosh\left(\sqrt{\frac{h_c \cdot per}{k_f \cdot A_c}} \cdot L_f\right) + \sqrt{\frac{h_c \cdot A_c}{k_f \cdot per}} \cdot \sinh\left(\sqrt{\frac{h_c \cdot per}{k_f \cdot A_c}} \cdot L_f\right)}$$

BC2: Specified temperature

$$q''_{B2}\left(h_c\right) := \sqrt{h_c \cdot per \cdot k_f \cdot A_c} \cdot \theta_B \cdot \frac{\cosh\left(\sqrt{\frac{h_c \cdot per}{k_f \cdot A_c}} \cdot L_f\right) - \frac{\theta_L}{\theta_B}}{\sinh\left(\sqrt{\frac{h_c \cdot per}{k_f \cdot A_c}} \cdot L_f\right)}$$

BC3: Insulated tip

$$q''_{B3}\left(h_c\right) := \sqrt{h_c \cdot per \cdot k_f \cdot A_c} \cdot \theta_B \cdot \tanh\left(\sqrt{\frac{h_c \cdot per}{k_f \cdot A_c}} \cdot L_f\right)$$

BC4: Infinitely long fin (for $L_f >> B_f$)

$$q''_{B4}\left(h_c\right) := \sqrt{h_c \cdot per \cdot k_f \cdot A_c} \cdot \theta_B$$

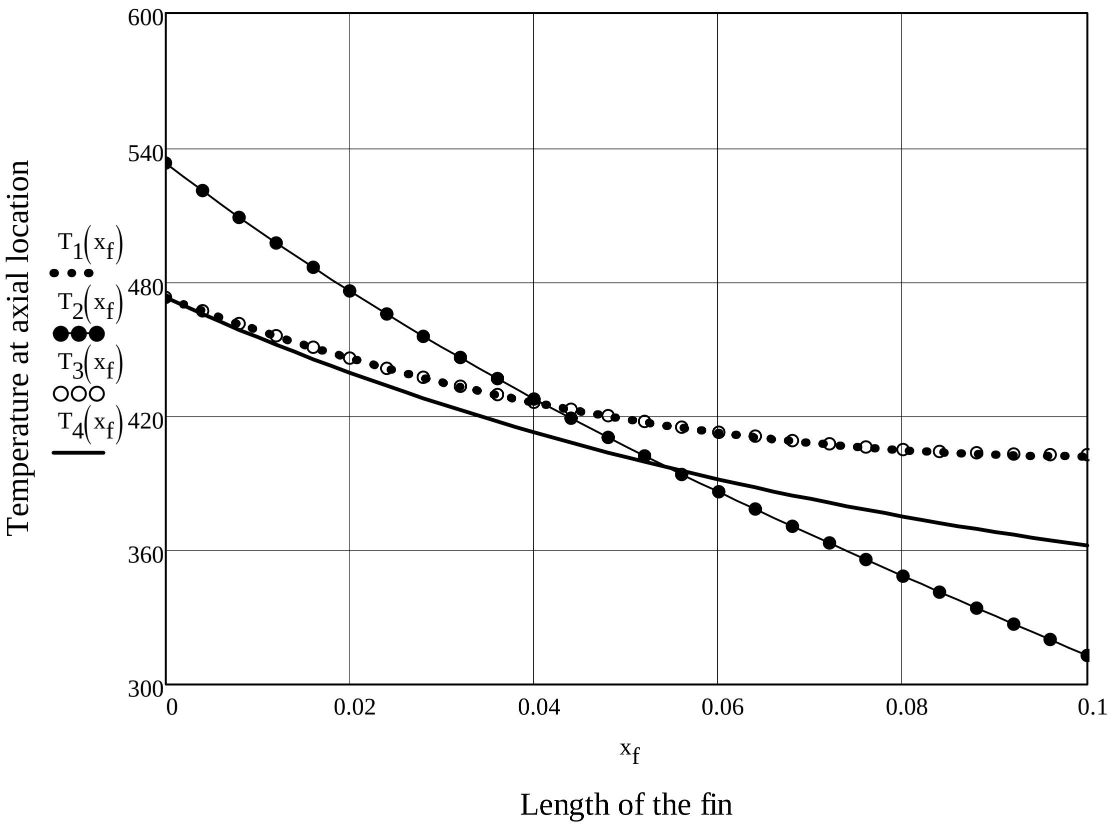
Heat flux through tip of the fin:

BC1: Specified convection $q''_{t1} := h_c \cdot A_c \cdot \theta_L$

BC2: Specified temperature $q''_{t2} := \dfrac{M \cdot (\theta_B + \theta_L)}{\sinh(m_f \cdot L_f)}$

BC3: Insulated tip $q''_{t3} := 0.0$

BC4: Infinitely long fin (for $L_f >> B_f$) $q''_{t4} := 0.0$



$\dfrac{h_c}{m_f \cdot k_f} = 0.008$ << 1.0

And hence, the temperature profiles for tip conditions "specified convection" and 'insulated' are same as one may have expected (shown by overlapping lines in the plot above).

$$h_c := 0, 1\frac{W}{m^2 \cdot K} .. 50\frac{W}{m^2 \cdot K}$$