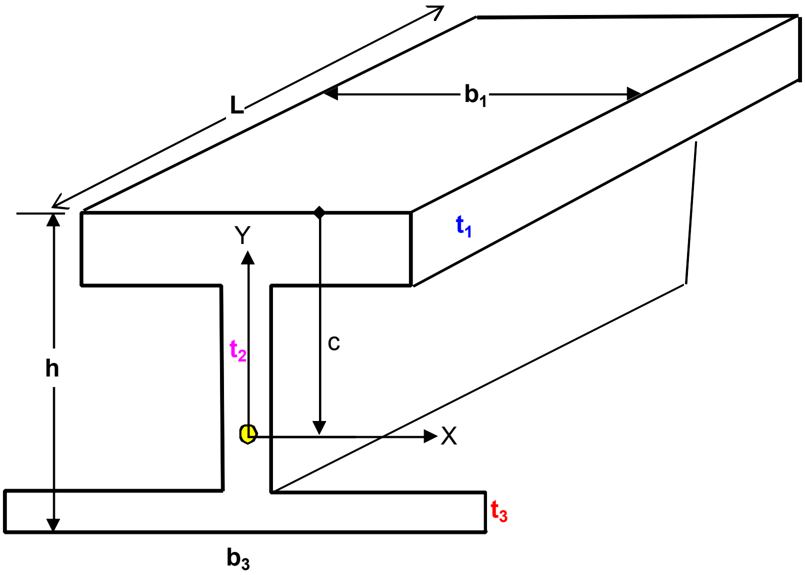
Stresses in I-Beams



$b_1 := 5.00in = 127 \cdot mm$

$b_3 := 8.00in = 0.203m$

$h_I := 6.00in = 152.4mm$

$t_1 := 1.00in = 25.4mm$

$t_2 := 1.00in = 25.4mm$

$t_3 := 1.00in = 25.4mm$

$L_I := 20.0in = 508 \cdot mm$

$F_{Ibeam} := 5000kgf$

$MB_I := 1000kgf \cdot m$

$\sigma_Y := 250MPa$

$\sigma_{UTS} := 350MPa$

$$c = \frac{b_1 \, . \, t_1^{\,2}/2 + [(h-t_3)^2 - t_1^{\,2}] \, . \, t_2/2 + b_3 \, . \, t_3 \, . \, (h-t_3/2)}{b_1 \, . \, t_1 + t_2 \, . \, [(h-t_3) - t_1] + b_3 \, . \, t_3}$$

$$I = b_1.t_1.\,[\,t_1^{\,2}/12 + (c - t_1/2)^2\,] \;\; + b_3.t_3.[\,t_3^{\,2}/12 + \{c-(h-t_3/2)\}^2]$$
$$t_2.[(h-t_3) - t_1].[\{(h - t_3) - t_1\}^2/12 + \{(h-t_3)+ t_1-2c\}^2/4]$$

Calculation of Shear Area, Area Moment of Inertia, Stresses and Factor of Safeties

$A_{Ibeam} := b_1 \cdot t_1 + (h_I - t_3 - t_1) \cdot t_2 + b_3 \cdot t_3 = 0.011m^2$ $\qquad A_{Ibeam} = 17 \cdot in^2$

$$c_I := \frac{b_1 \cdot t_1^{\,2} + \left[(h_I - t_3)^2 - t_1^{\,2}\right] \cdot t_2 + b_3 \cdot t_3 \cdot (2 \cdot h_I - t_3)}{2A_{Ibeam}} = 87.4mm \qquad c_I = 3.44 \cdot in$$

$$MI_{Ibeam}(b_1, b_3, h_I, t_1, t_2, t_3) := \begin{vmatrix} x_1 \leftarrow b_1 \cdot t_1 \cdot \left[\dfrac{t_1^{\,2}}{12} + \left(c_I - \dfrac{t_1}{2}\right)^2\right] + b_3 \cdot t_3 \cdot \left[\dfrac{t_3^{\,2}}{12} + \left[c_I - \left(h_I - \dfrac{t_3}{2}\right)\right]^2\right] \\ x_1 + t_2 \cdot (h_I - t_1) \cdot \left[\dfrac{(h_I - t_3 - t_1)^2}{12} + \dfrac{(h_I - t_3 + t_1 - 2 \cdot c_I)^2}{4}\right] \end{vmatrix}$$

$I_{Ibeam} := MI_{Ibeam}(b_1, b_3, h_I, t_1, t_2, t_3) = 3.575 \times 10^{-5} m^4$ $\qquad I_{Ibeam} = 85.886 \cdot in^4$

Note the direction of X-Axis

Shear Stress: $\qquad \tau_{Ibeam} := \dfrac{F_{Ibeam}}{A_{Ibeam}} = 4.471 \cdot MPa$

Bending Stress - Top Fiber: $\qquad \sigma_{I.Top} := \dfrac{MB_I}{I_{Ibeam}} \cdot c_I = 24 \cdot MPa$

Bending Stress - Bottom Fiber: $\sigma_{I.Bot} := \frac{MB_I}{I_{Ibeam}} \cdot (c_I - h_I) = -18 \cdot MPa$

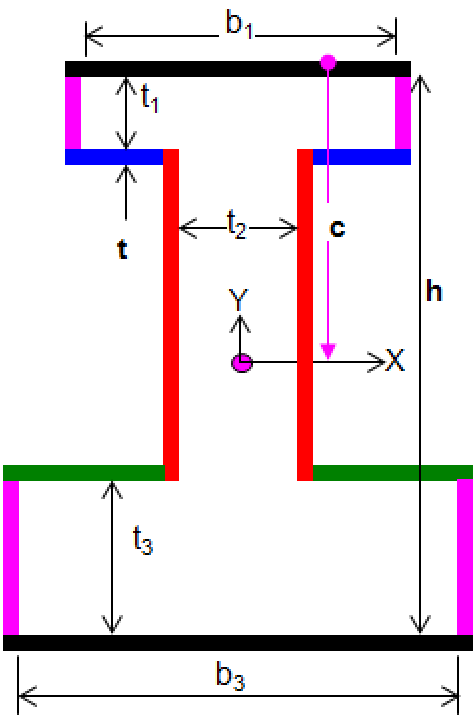
Maximum Shear Stress: $\tau_{T.Max} := \left(\frac{\max(|\sigma_{I.Top}|, |\sigma_{I.Bot}|)^2}{4} + \tau_{Ibeam}^{\;2} \right)^{0.5} = 13 \cdot MPa$

von Mises Stress: $\sigma_{e.Ibeam} := \left(\max(|\sigma_{I.Top}|, |\sigma_{I.Bot}|)^2 + 3\tau_{Ibeam}^{\;2} \right)^{0.5} = 25 \cdot MPa$

Factor of Safeties: $FOS_{I.YLD} := \frac{\sigma_Y}{\sigma_{e.Ibeam}} = 9.92$ **OK if > 2.0**

$FOS_{I.UTS} := \frac{\sigma_{UTS}}{\sigma_{e.Ibeam}} = 13.89$ **OK if > 3.0**

Welds in a I-Beam



A = $(b_1 + b_3 - t_2) * 2 * t + (h + 2*t) * 2*t$

c = $(b_1+2t)\, t^2/2 + 2.\, t.\, t_1.\, (t + t_1/2) + 2\, .\, t.\, t_3.\, (h+t-t_3/2)+$
$(b_1-t_2)\, t.\, (3t/2 + t_1) + (h-t1-t3)\, 2t.\, (h + 3t/2 - t_3) +$
$(b_3-t_2)\, t.\, (h + t/2 - t_3) + (b_3 + 2t)\, .\, t.\, (h + 3t/2)$

Note: dimension 'c' is with respect to top face

$I_{xx} =$ $t.\, (b_1+2t)\, .[\, t^2/12 + (c-t/2)^2\,] +$
$2t.t_1.\, [t_1^{\,2}/12+(c-t-t_1/2)^2] +$
$t.\, (b_1 - t_2).\, [t^2/12 + (c-t_1-3t/2)^2] +$
$t.(b3- t2).[t^2/12 +(h+t/2-t_3-c)^2]$
$2t.(h-t_1-t_3).[(h-t_1-t_3)^2/12 + \{c-t-(h+t_1-t_3)/2\}^2]+$
$2t_3.\, t.\, [t_3^{\,2}/12 + (h+t - t_3/2-c)^2] +$
$t.(b_3+2t).\, [\, t^2/12 + (h+3t/2-c)^2\,]$

Weld Leg Length

$t_{Leg} := 0.5in$

Weld Throat Length:

$t_w := \frac{t_{Leg}}{\sqrt{2}} = 0.354 \cdot in$

Weld Joint Efficiency:

$\eta_w := 75\%$

Calculation of Shear Area, Area Moment of Inertia, Stresses and Factor of Safeties

$A_{I.Weld}(t_w) := A_1 \leftarrow (b_1 + b_3 - t_2) \cdot 2t_w + (h_I + 2t_w) \cdot 2t_w$

$A_{I.Weld}(t_w) = 8.534 \times 10^3 \cdot mm^2$

$$c_{I.Weld}(t_w) := \begin{array}{|l} x_1 \leftarrow (b_1 + 2\cdot t_w)\cdot\frac{t_w^{\,2}}{2} + 2\cdot t_w\cdot t_1\cdot\left(t_w + \frac{t_1}{2}\right) + t_w\cdot(b_3 + 2\cdot t_w)\cdot\left(\frac{3\cdot t_w}{2} + h_I\right) \\ x_2 \leftarrow 2\cdot t_w\cdot t_3\cdot\left(h_I + t_w - \frac{t_3}{2}\right) + t_w\cdot(b_1 - t_2)\cdot\left(t_1 + \frac{3t_w}{2}\right) + (h_I - t_1 - t_3)\cdot 2\cdot t_w\cdot\left(h_I + \frac{3\cdot t_w}{2} - t_3\right) \\ \dfrac{x_1 + x_2 + (b_3 - t_2)\cdot t_w\cdot\left(h_I + \frac{t_w}{2} - t_3\right)}{A_{I.Weld}(t_w)} \end{array}$$

$$c_{I.Weld}(t_w) = 107.2\cdot mm$$

$$MI_{I.Weld}(t_w) := \begin{array}{|l} x_1 \leftarrow t_w\cdot(b_1 + 2\cdot t_w)\cdot\left[\frac{t_w^{\,2}}{12} + \left(c_{I.Weld}(t_w) - \frac{t_w}{2}\right)^2\right] \\ x_2 \leftarrow x_1 + 2\cdot(t_w\cdot t_1)\cdot\left[\frac{t_1^{\,2}}{12} + \left(c_{I.Weld}(t_w) - t_w - \frac{t_1}{2}\right)^2\right] \\ x_3 \leftarrow x_2 + t_w\cdot(b_1 - t_2)\cdot\left[\frac{t_w^{\,2}}{12} + \left(c_{I.Weld}(t_w) - \frac{3\cdot t_w}{2} - t_1\right)^2\right] \\ x_4 \leftarrow x_3 + t_w\cdot(b_3 - t_2)\left[\frac{t_w^{\,2}}{12} + \left(h_I + \frac{t_w}{2} - t_3 - c_{I.Weld}(t_w)\right)^2\right] \\ x_5 \leftarrow x_4 + 2t_w\cdot t_3\cdot\left[\frac{t_3^{\,2}}{12} + \left(h_I + t_w - \frac{t_3}{2} - c_{I.Weld}(t_w)\right)^2\right] \\ x_6 \leftarrow x_5 + 2\cdot t_w\cdot(h_I - t_1 - t_3)\cdot\left[\frac{(h_I - t_1 - t_3)^2}{12} + \left(c_{I.Weld}(t_w) - t_w - \frac{h_I + t_1 - t_3}{2}\right)^2\right] \\ x_6 + t_w\cdot(b_3 + 2\cdot t_w)\cdot\left[\frac{t_w^{\,2}}{12} + \left(h_I + \frac{3\cdot t_w}{2} - c_{I.Weld}(t_w)\right)^2\right] \end{array}$$

$$MI_{I.Weld}(t_w) = 3.243 \times 10^{-5}\, m^4$$

Shear Stress: $\tau_{Ibeam.W} := \dfrac{F_{Ibeam}}{A_{I.Weld}(t_w)} = 5.746\cdot MPa$

Bending Stress-Top Fiber: $\sigma_{I.W.Top} := \dfrac{MB_I}{MI_{I.Weld}(t_w)}\cdot c_{I.Weld}(t_w) = 32\cdot MPa$

Bending Stress - Bottom: $\sigma_{I.W.Bot} := \dfrac{MB_I}{MI_{I.Weld}(t_w)}\cdot\left(c_{I.Weld}(t_w) - h_I\right) = -14\cdot MPa$

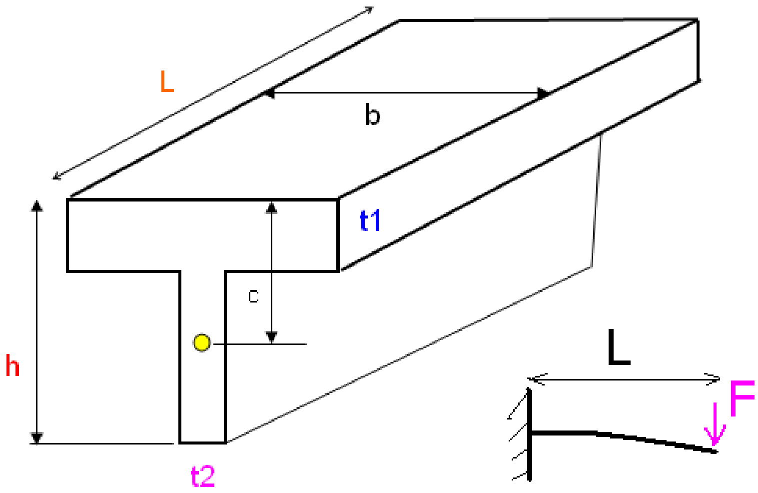
Maximum Shear Stress: $\tau_{I.Weld.Max} := \left(\dfrac{\max\left(\left|\sigma_{I.W.Top}\right|, \left|\sigma_{I.W.Bot}\right|\right)^2}{4} + \tau_{Ibeam.W}^{\,2}\right)^{0.5} = 17\cdot MPa$

von Mises Stress: $\sigma_{e.Ibeam.Weld} := \left(\max\left(\left|\sigma_{I.W.Top}\right|, \left|\sigma_{I.W.Bot}\right|\right)^2 + 3\tau_{Ibeam.W}^{\,2}\right)^{0.5} = 34\cdot MPa$

Factor of Safeties: $FOS_{I.Weld.Y} := \dfrac{\sigma_Y}{\sigma_{e.Ibeam.Weld}}\cdot\eta_w = 5.53$ **OK if > 2.0**

$FOS_{T.Weld.U} := \dfrac{\sigma_{UTS}}{\sigma_{e.Ibeam.Weld}}\cdot\eta_w = 7.7$ **OK if > 3.0**

Stresses in Tee-Beams



$b_T := 5.00 in = 127 \cdot mm$

$h_T := 6.00 in = 152.4 mm$

$t_1 := 0.5 in = 12.7 \cdot mm$

$t_2 := 1.00 in = 25.4 mm$

$L_T := 25.0 in = 635 \cdot mm$

$F_T := 5000 kgf$

$MB_T := 2000 kgf \cdot m$

$\sigma_Y := 250 MPa$

$\sigma_{UTS} := 400 MPa$

$$c = \frac{b \,.\, t_1^2/2 + (h^2 - t_1^2) \,.\, t_2/2}{b \,.\, t_1 + t_2 \,.\, (h - t_1)}$$

$$I = b \,.\, t_1 \,.\, (t_1^2/12 + (c - t_1/2)^2) + t_2 .(h - t_1).[(h - t_1)^2/12 + (h + t_1 - 2c)^2/4]$$

$$c_T := \frac{\frac{b_T \cdot t_1^2}{2} + \frac{\left(h_T^2 - t_1^2\right) \cdot t_2}{2}}{b_T \cdot t_1 + t_2 \cdot \left(h_T - t_1\right)} = 58.7 \cdot mm$$

Calculation of Shear Area, Area Moment of Inertia, Stresses and Factor of Safeties

$A_T := b_T \cdot t_1 + \left(h_T - t_1\right) \cdot t_2 = 5.161 \times 10^{-3} m^2$ $\quad A_T = 8 \cdot in^2$ $\quad c_T = 2.31 \cdot in$

$$MI_T\left(b_T, h_T, t_1, t_2\right) := \left| \begin{array}{l} x_1 \leftarrow b_T \cdot t_1 \cdot \left[\frac{t_1^2}{12} + \left(c_T - \frac{t_1}{2}\right)^2 \right] \\ x_1 + t_2 \cdot \left(h_T - t_1\right) \cdot \left[\frac{\left(h_T - t_1\right)^2}{12} + \frac{\left(h_T + t_1 - 2 \cdot c_T\right)^2}{4} \right] \end{array} \right.$$

$I_{Tee} := MI_T\left(b_T, h_T, t_1, t_2\right) = 1.223 \times 10^{-5} m^4$ $\quad I_{Tee} = 29.385 \cdot in^4$

Note the direction of X-Axis

Shear Stress: $\tau_{Tee} := \frac{F_T}{A_T} = 9.5 \cdot MPa$

Bending Stress - Top Fiber: $\sigma_{T.Top} := \frac{MB_T}{I_{Tee}} \cdot c_T = 94 \cdot MPa$

Bending Stress - Bottom Fiber: $\sigma_{T.Bot} := \frac{MB_T}{I_{Tee}} \cdot \left(c_T - h_T\right) = -150 \cdot MPa$

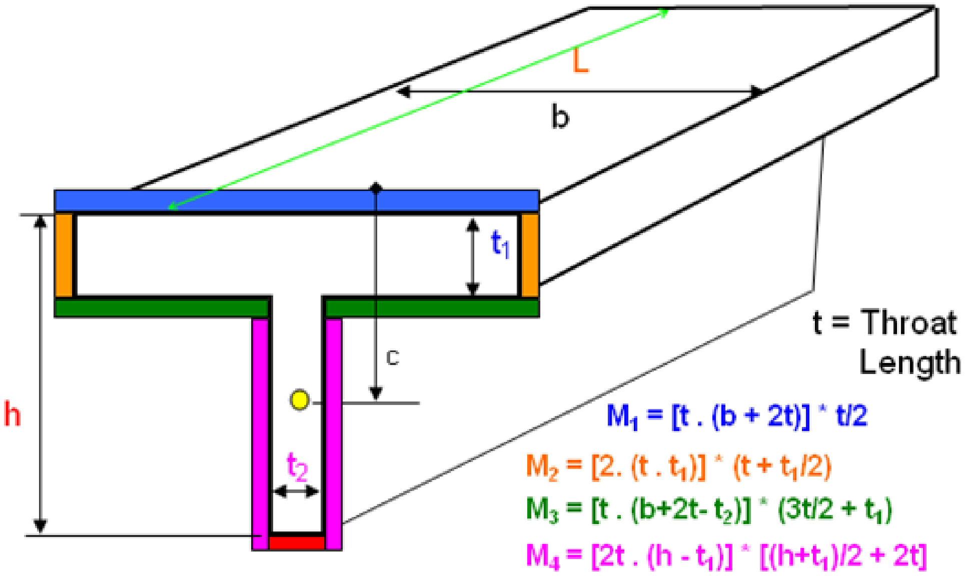
Maximum Shear Stress: $\tau_{T.Max} := \left(\frac{\max\left(\left|\sigma_{T.Top}\right|, \left|\sigma_{T.Bot}\right|\right)^2}{4} + \tau_{Tee}^2\right)^{0.5} = 76 \cdot MPa$

von Mises Stress: $\sigma_{e.Tee} := \left(\max\left(\left| \sigma_{T.Top} \right| , \left| \sigma_{T.Bot} \right| \right)^2 + 3\tau_{Tee}{}^2 \right)^{0.5} = 151 \cdot MPa$

Factor of Safeties: $FOS_{T.YLD} := \dfrac{\sigma_Y}{\sigma_{e.Tee}} = 1.65$ **OK if > 2.0**

$FOS_{T.UTS} := \dfrac{\sigma_{UTS}}{\sigma_{e.Tee}} = 2.65$ **OK if > 3.0**

Welds in a Tee-Beam



$M_1 = [t \,.\, (b + 2t)] * t/2$

$M_2 = [2. \,(t \,.\, t_1)] * (t + t_1/2)$

$M_3 = [t \,.\, (b+2t- t_2)] * (3t/2 + t_1)$

$M_4 = [2t \,.\, (h - t_1)] * [(h+t_1)/2 + 2t]$

$M_5 = [t \,.\, t_2] * (h + 3t\,/\,2)$

$$c = \frac{M_1 + M_2 + M_3 + M_4 + M_5}{[(b+2t) \,.\, t] + 2*[t_1 \,.\, t] + [(b+2t-t_2) \,.\, t] + 2*[(h-t_1) \,.\, t] + t_2 \,.\, t}$$

$$\begin{aligned} I = \; & [t \,.(b+2t)] \,.\, [t^2/12 + (c - t/2)^2\,] + 2*(t \,.\, t_1).[t_1{}^2/12 + (c - t - t_1/2)^2\,] + \\ & t \,.\, (b+2t-t_2) \,.\, [t^2/12 + (c - 3t/2 - t_1)^2\,] + \\ & 2*[t \,.\, (h-t_1)] \,.\, [(h-t_1)^2/12 + \{(h+t_1)/2 + 2t - c\}^2] + \\ & (t \,.\, t_2) \,.\, [t^2/12 + (h + 3t/2 - c)^2\,] \end{aligned}$$

Weld Leg Length

$t_{Leg} := 0.5in$

Weld Throat Length:

$t_w := \dfrac{t_{Leg}}{\sqrt{2}} = 0.354 \cdot in$

Weld Joint Efficiency:

$\eta_w := 75\%$

Calculation of Shear Area, Area Moment of Inertia, Stresses and Factor of Safeties

$$A_{T.Weld}(t_w) := \left| \begin{array}{l} A_1 \leftarrow (b_T + 2 \cdot t_w) \cdot t_w + 2 \cdot t_1 \cdot t_w + t_2 \cdot t_w \\ A_1 + \left[(b_T + 2 \cdot t_w - t_2) \cdot t_w + 2 \cdot (h_T - t_1) \cdot t_w \right] \end{array} \right.$$

$A_{T.Weld}(t_w) = 5.341 \times 10^3 \cdot mm^2$

$$c_{T.Weld}(t_w) := \left| \begin{array}{l} x_1 \leftarrow t_w \cdot (b_T + 2 \cdot t_w) \cdot \frac{t_w}{2} + 2 \cdot t_w \cdot t_1 \cdot \left(t_w + \frac{t_1}{2} \right) + t_w \cdot (b_T + 2 \cdot t_w - t_2) \cdot \left(\frac{3 \cdot t_w}{2} + t_1 \right) \\ x_2 \leftarrow 2 \cdot t_w \cdot (h_T - t_1) \left[\frac{(h_T + t_1)}{2} + 2t_w \right] + t_w \cdot t_2 \cdot \left(h_T + \frac{3t_w}{2} \right) \\ \frac{x_1 + x_2}{A_{T.Weld}(t_w)} \end{array} \right.$$

$$c_{T.Weld}(t_w) = 61.3 \cdot mm$$

$$I_{T.Weld}(t_w) := \left| \begin{array}{l} x_1 \leftarrow t_w \cdot (b_T + 2 \cdot t_w) \cdot \left[\frac{t_w^2}{12} + \left(c_{T.Weld}(t_w) - \frac{t_w}{2} \right)^2 \right] \\ x_2 \leftarrow x_1 + 2 \cdot (t_w \cdot t_1) \cdot \left[\frac{t_1^2}{12} + \left(c_{T.Weld}(t_w) - t_w - \frac{t_1}{2} \right)^2 \right] \\ x_3 \leftarrow x_2 + t_w \cdot (b_T + 2 \cdot t_w - t_2) \cdot \left[\frac{t_w^2}{12} + \left(c_{T.Weld}(t_w) - \frac{3 \cdot t_w}{2} - t_1 \right)^2 \right] \\ x_4 \leftarrow x_3 + 2 \cdot t_w \cdot (h_T - t_1) \left[\frac{(h_T - t_1)^2}{12} + \left(\frac{h_T + t_1}{2} + 2 \cdot t_w - c_{T.Weld}(t_w) \right)^2 \right] \\ x_4 + t_w \cdot t_2 \cdot \left[\frac{t_w^2}{12} + \left(h_T + \frac{3 \cdot t_w}{2} - c_{T.Weld}(t_w) \right)^2 \right] \end{array} \right.$$

$$I_{T.Weld}(t_w) = 39.549 \cdot in^4$$

Shear Stress: $\tau_{Tee.Weld} := \frac{F_T}{A_{T.Weld}(t_w)} = 9.181 \cdot MPa$

Bending Stress - Top Fiber: $\sigma_{T.W.Top} := \frac{MB_T}{I_{T.Weld}(t_w)} \cdot c_{T.Weld}(t_w) = 73 \cdot MPa$

Bending Stress - Bottom: $\sigma_{T.W.Bot} := \frac{MB_T}{I_{T.Weld}(t_w)} \cdot (c_{T.Weld}(t_w) - h_T) = -109 \cdot MPa$

Maximum Shear Stress: $\tau_{T.Weld.Max} := \left(\frac{\max(|\sigma_{T.W.Top}|, |\sigma_{T.W.Bot}|)^2}{4} + \tau_{Tee.Weld}^2 \right)^{0.5} = 55 \cdot MPa$

von Mises Stress: $\sigma_{e.Tee.Weld} := \left(\max(|\sigma_{T.W.Top}|, |\sigma_{T.W.Bot}|)^2 + 3\tau_{Tee.Weld}^2 \right)^{0.5} = 110 \cdot MPa$

Factor of Safeties: $FOS_{T.Weld.Y} := \frac{\sigma_Y}{\sigma_{e.Tee.Weld}} \cdot \eta_w = 1.71$ **OK if > 2.0**

$FOS_{T.Weld.U} := \frac{\sigma_{UTS}}{\sigma_{e.Tee.Weld}} \cdot \eta_w = 2.7$ **OK if > 3.0**