1.9 Fan Laws – Only One Row of Rotating Blades

This configuration refers to a configuration where no Inlet Guide Vane (IGV) or Exit Guide Vane (EGV) is present. The system consists of just the rotating blades and shroud to control the flow conditions near the blade tip. Other performance enhancing features such as bell-mouth entry and exit may be present.

1.9.1 Assumptions

1. Effect of Reynolds Number is negligible (which it often is but not always)
2. Machines are "geometrically similar" (one can be obtained by uniform enlargement or reduction of shape).
3. These machines are operating under similar conditions so that dynamic similarity applies that is $\phi_1 = \phi_2, \psi_1 = \psi_2, \Lambda_1 = \Lambda_2, \eta_1 = \eta_2$

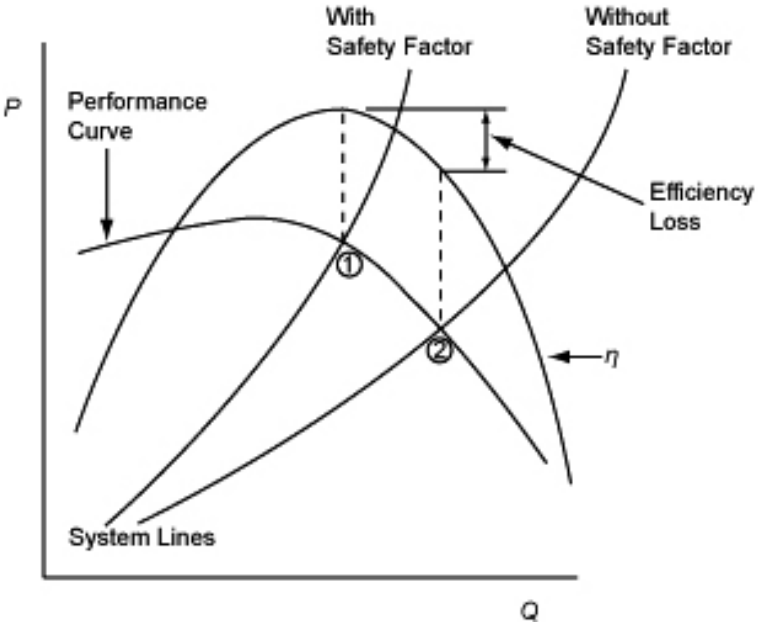
1.9.2 Independent Variables

1. Characteristic length dimension (mean diameter, D or blade height, H_B or chord length, L_C)
2. Characteristic Flow Area (annulus passage formed by blade tip and hub diameters)
3. Rotation Speed (blade tangential speed), ω
4. Fluid Viscosity (defines geometric and kinematic similarity)
5. Fluid Density (defines dynamic similarity), ρ
6. Surface Roughness (defines friction losses), ε
7. Number of blades in a row (solidity), N_B

1.9.3 Dependent Variables

1. Head Rise, H or $\Delta p/\rho$
2. Through Flow, Q
3. Shaft power required to operate the fan, W
4. Efficiency (Static or Total as per arrangement of fan), η

1.9.4 The Operating and the Surge Point



1.9.4.1 Surge Point or Surge Limit

This is the point of maximum pressure or minimum flow where the developed pressure is inefficient to overcome the system / machine resistance. When surge occurs, gas in discharge piping will flow back into the compressor. With no discharge flow, the discharge pressure drops further and the cycle repeats itself which may ultimately cause compressor blades to overheat.

1.9.4.2 Chocking

This is the point of maximum flow and the minimum pressure where flow through the compressor cannot increase any further without further modification in the compressor. This phenomena occurs when the gas reaches the sonic velocity (velocity of sound in the working fluid and location pressure and temperature conditions) at any point in the compressor

1.9.5 Performance Scaling Laws

- Speed has a linear relationship with airflow, a second-order relationship with pressure, and a third-order relationship with power.
- Static pressure increase as the square of the volume flow rate
- Velocity pressure also increase as the square of the volume flow rate
- Since the total pressure is a linear sum of static and velocity pressure, total pressure will also vary as square of the volume flow rate.
- Volume flow increase directly in proportion to the rotation speed

- Static pressure also varies directly with the density
- Shaft power is proportion to the product of volume flow rate and total pressure, the shaft power varies cube of the rotation speed and proportional to the density.

S. No.	Constant	Variable	Fan Scaling (Affinity) Law
1	D_{MEAN}, σ **[$D_1 = D_2$]**	ω (speed), ρ (density)	$Q_2 = Q_1\left(\frac{\omega_2}{\omega_1}\right)$ .....(1A) $\Delta p_2 = \Delta p_1\left[\frac{\omega_2}{\omega_1}\right]^2 \frac{\rho_2}{\rho_1}$ .....(1B) $\dot{W}_2 = \dot{W}\left\{\frac{\omega_2}{\omega_1}\right\}^3 \frac{\rho_2}{\rho_1}$ .....(1C) $\dot{m}_2 = \dot{m}_1\left(\frac{\omega_2}{\omega_1}\right)\frac{\rho_2}{\rho_1}$ .....(1D)
2	ω, σ **[$\omega_1 = \omega_2$]**	D_{MEAN}, ρ (density)	$Q_2 = Q_1\left(\frac{D_2}{D_1}\right)^3$ .....(2A) $\Delta p_2 = \Delta p_1\left[\frac{D_2}{D_1}\right]^2 \frac{\rho_2}{\rho_1}$ .....(2B) $\dot{W}_2 = \dot{W}\left\{\frac{D_2}{D_1}\right\}^5 \frac{\rho_2}{\rho_1}$ .....(2C) $\dot{m}_2 = \dot{m}_1\left(\frac{D_2}{D_1}\right)^3 \frac{\rho_2}{\rho_1}$ .....(2D)
3	D, ω, Q	σ	No such correlation exist to account for change in solidity

In General: $Q_2 = Q_1\left(\frac{\omega_2}{\omega_1}\right)\left[\frac{D_2}{D_1}\right]^3$ --------- [1.1] $\Delta p_2 = \Delta p_1\left[\frac{\omega_2}{\omega_1}\right]^2\left(\frac{D_2}{D_1}\right)^2 \frac{\rho_2}{\rho_1}$ .....

[1.2]

$$\dot{W}_2 = \dot{W}\left\{\frac{\omega_2}{\omega_1}\right\}^3\left[\frac{D_2}{D_1}\right]^5 \frac{\rho_2}{\rho_1} \text{ ... [1.3]}$$

From [1.2], $\left[\frac{\omega_2}{\omega_1}\right] = \left(\frac{D_1}{D_2}\right)\left[\frac{\Delta p_2}{\Delta p_1}\right]\left[\frac{\rho_1}{\rho_2}\right]^{1/2}$ ----- [2.1]

From [1.1] and [2.1], $$Q_2 = Q_1 \left(\frac{\Delta p_2}{\Delta p_1} \right)^{1/2} \left[\frac{D_2}{D_1} \right]^2 \left[\frac{\rho_1}{\rho_2} \right]^{1/2}$$ ----- [2.2]

From [1.3 and [2.1], $$\dot{W}_2 = \dot{W} \left\{ \frac{\Delta p_2}{\Delta p_1} \right\}^{3/2} \left[\frac{D_2}{D_1} \right]^2 \left(\frac{\rho_1}{\rho_2} \right)^{1/2}$$ ----- [2.3]

Few graphical representations of fan laws are as follows:

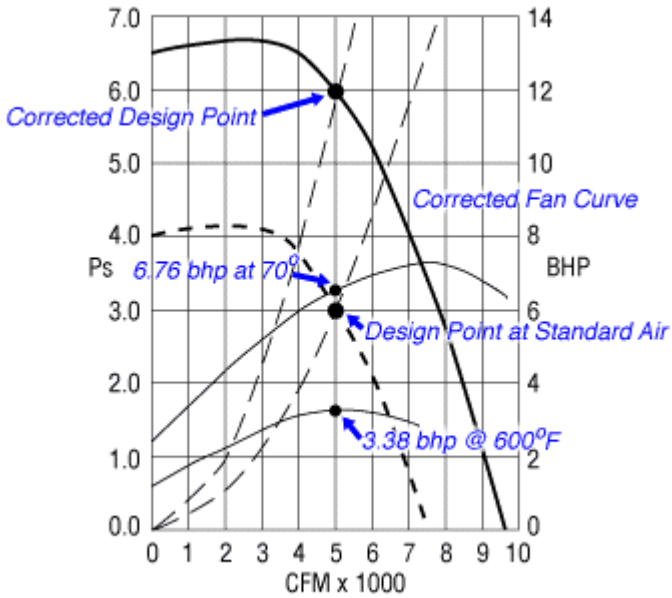


Figure 1-10: Effect of change in gas density on Fan Curve (Ref: greenheck.com)

From [1.1], $$\omega_2 = \omega_1 \left(\frac{Q_2}{Q_1} \right) \left[\frac{D_1}{D_2} \right]^3$$ ----- [3.1]

From [1.2] and [3.1] or from [2.2], $$\Delta p_2 = \Delta p_1 \left(\frac{D_1}{D_2} \right)^4 \left[\frac{Q_2}{Q_1} \right]^2 \frac{\rho_2}{\rho_1}$$ ----- [3.2]

From [2.3], $$\dot{W}_2 = \dot{W} \left\{ \frac{Q_2}{Q_1} \right\}^3 \left[\frac{D_1}{D_2} \right]^4 \left(\frac{\rho_2}{\rho_1} \right)$$ ----- [3.3]

1.9.6 Sound power

$$L_2 = L_1 + 55 \cdot \left[\log_{10} \left(\frac{\omega_2}{\omega_1} \right) + \log_{10} \left(\frac{D_2}{D_1} \right) + \log_{10} \left(\frac{\rho_2}{\rho_1} \right) \right]$$

1.10 Examples of Fan Scaling

1.10.1 Change of Fluid

Many a times the prototype fan is tested on air under atmospheric pressure conditions and displaces different fluid in actual operations such as exhaust fumes or gas mixtures (CO + H_2: the gasification of coal, H_2+N_2: cooling media in turbo-generators) at different pressure and temperature conditions.

Let's define a sign convention:

ρ_A = Density of air under ***p***rototype or test conditions

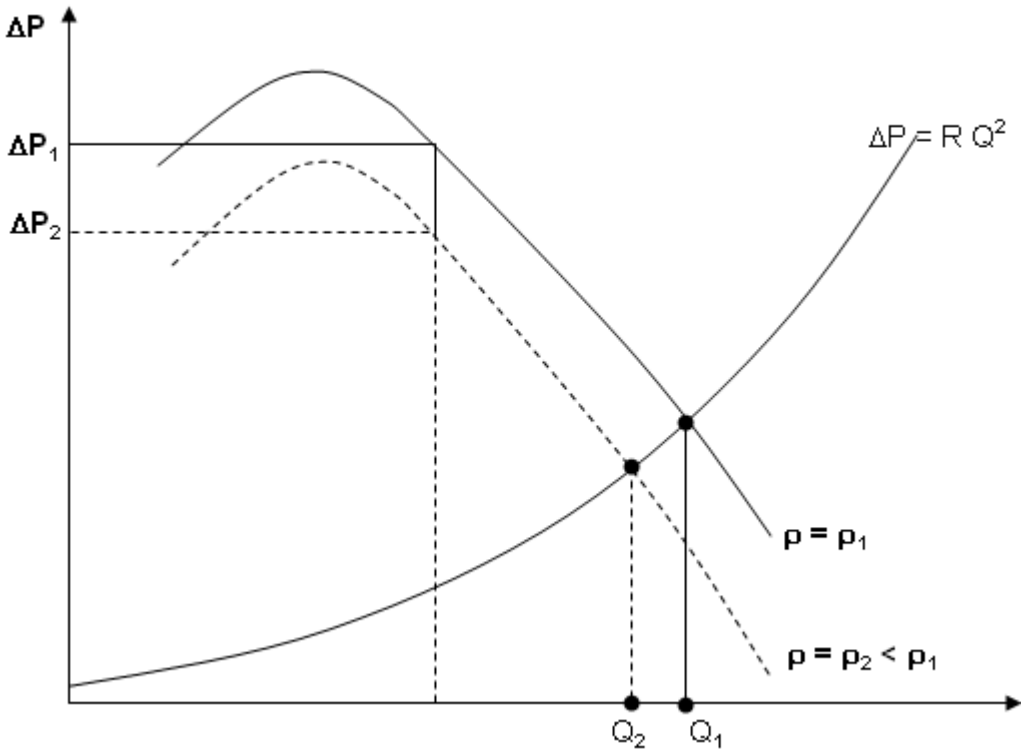
ρ_H = Density of hydrogen at ***a***ctual pressure and temperature conditions

Let us further assume that there is no change in fan dimensions, that is, hub diameters, blade diameters, chord width and tip clearance are identical.

- From equation [1A], volume flow rate though the blower remains same for both the fluid conditions.
- From equation [1B] above, $\Delta p_H = \Delta p_A \left[\frac{\omega_H}{\omega_A}\right]^2 \frac{\rho_H}{\rho_A}$
- In case the rotation speed remains same, $\Delta p_H = \Delta p_A \frac{\rho_H}{\rho_A}$
- Thus, for the equal volume flow rate, the 'static' pressure rise across the fan increase or decrease in the proportion of increase or decrease in density.
- Equation [3.2] can be used to account for both the change in volume flow rate and the density of the working fluid.

1.10.2 Change of Fluid Operating Pressure / Density

A fan operating at a higher elevation or temperature will move the SAME VOLUME of air as it will at standard conditions. However, it will generate LESS TOTAL PRESSURE and will require LESS HORSEPOWER.


ΔP
ΔP1
ΔP2
ΔP = R Q2
ρ = ρ1
ρ = ρ2 < ρ1
Q2
Q1