Due to the dependence on 3 spatial coordinates, 2 local direction coordinates, s, and frequency, formal solution of the radiative transfer equation is very time consuming and usually accomplished by the use of approximate models for the directional and spectral dependencies. For directional approximations, CFX includes Rosseland, P-1, Discrete Transfer & Monte Carlo. For spectral approximations, it includes: Gray, Multiband and Weighted Sum of Gray Gases.

**IRRADIANCE (W/m²):**  
H: This is the flux of energy that Irradiates a surface, i.e. total radiation falling on the surface. In CFX Post, Wall Irradiation Flux represents the incoming radiative flux. It is computed as the solid angle integral of the incoming Radiative Intensity over a hemisphere on the boundary. For simulations using multiband model, the wall Irradiation Flux for each spectral band is also available for post-processing.

**RADIOSITY (W/m²):**  
B: Total flux of RADIATIVE energy away from a surface = \( \rho \cdot H - \varepsilon \cdot e_b \)  
Radiosity = Fraction of irradiated energy reflected by the surface + Radiative Energy emitted by it

**Heat Flux leaving any Particular Surface: (W/m²)**  
\( q = B - H \)

**Wall Radiative Heat Flux:**  
It represents the net radiative energy flux leaving the boundary. It is computed as the difference between the radiative emission and the incoming radiative flux

**Wall Heat Flux:**  
This is sum of the Wall Radiative Heat Flux and the Wall Convective Heat Flux. For an adiabatic wall, the sum should be zero.

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**Radiation Models in CFX:**

<table>
<thead>
<tr>
<th>Model</th>
<th>Description</th>
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<td>1 Rosseland Model</td>
<td>Diffusion Approximation Model</td>
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<td>Thermally Dense Medium, Does not solve any additional transport equation</td>
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<tr>
<td>2 P1 Model or Gibb's Model or Spherical Harmonics Model</td>
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<tr>
<td>3 Discrete Transfer Model or Ray-Tracing Method:</td>
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**Radiation Terminology**

**Absorptivity**  
Refers to the fraction of incoming energy that is absorbed at the surface.

**Diffuse**  
Refers to quantities that do not depend on incoming or outgoing direction; however, they might be functions of temperature as well as locations.

**Gray**  
Refers to quantities that do not have spectral dependency, i.e., they aren't function of frequency, wavelength or wavenumber.

**Opaque**  
Refers to a surface through which radiation cannot travel, i.e., radiation is reflected and/or absorbed at the surface.

**Reflectivity**  
Refers to the fraction of incoming energy that is reflected at a surface. It is a function of direction and frequency.
Spectral
Refers to a quantity that is a function of any of the spectrum variables: frequency, wavelength and wavenumber. The preferred variable is frequency but to avoid inconsistencies when dealing with non-unitary refractive index materials, wavelength or wavenumber in vacuum are also supported.

Transmissivity
Transmissivity, $\tau$, refers to the fraction of incoming energy that travels through the surface, i.e., the surface is semi-transparent when $0 < \tau < 1$, transparent when $\tau = 1$ and opaque when $\tau = 0$.

Radiative Transfer Equation: (Analogous to Navier-Stokes Equation for Convection)
1. RTE determines Intensity Radiation Field
2. Intensity field is further used to perform radiative energy balance over surfaces and volumes
3. RTE involves both Differentiation and Integration. Hence, very few analytical solutions exist.

Simplification of RTE:
1. Thermally transparent media, i.e. radiation heat exchange takes place between surfaces only.
2. Radiosity-Irradiocity Method is one such simplified method where RTE is not solved directly

\[ \frac{dI(\hat{s})}{d\ell} = - (\kappa + \sigma_s) I(\hat{s}) + \kappa I_b + \frac{\sigma_s}{4\pi} \int \phi(\hat{s}'; \hat{s}) I(\hat{s}') d\Omega' \]

The first term, which is negative, accounts for the decrease of the number of photons on the given direction, either because it is absorbed by the medium (with an absorption coefficient $\kappa$) or because it is scattered onto another direction (with a scattering coefficient $\sigma_s$).

The second term has positive sign, therefore implying an increase of the number of photons. This term is due to thermal emission of photons. It is zero only if the temperature is zero, or the absorption coefficient is zero. Notice that the proportionality coefficient is the same absorption coefficient appearing in the first term. This holds under the assumption of local thermodynamic equilibrium. $I_b$ is blackbody emission intensity.

The third term contributes only if the medium scatters radiation. $F$ is in-scattering phase function.

Example:
There are already a number of methods for computing radiative heat transfer. Most widely applied method in glass industry is undoubtedly the Rosseland Approximation, which is also known as the Diffusion Approximation. Rosseland, an Astrophysicist, derived this method while studying radiative transfer within a star.

The first assumption is that of so-called optical thickness. This means that the glass object should be geometrically thick or heavily absorbing infrared light. For glass furnaces and for dark glasses in feeder channels this assumption is valid, for many other applications it is not.

The Rosseland Approximation further assumes that accuracy is not needed close to any boundary. Unfortunately, this is often exactly the region where one would like to know the temperature distribution accurately, as it is typically the region with the steepest temperature gradients.

Unfortunately, in glass production most processes are neither optically thick nor optically thin, but rather somewhere in between. This means that above simple approximative methods
cannot be used. In that case, we cannot but solve the Radiate Transfer Equation (RTE), which is the equation that describes how the radiation changes within the domain. The RTE is a partial differential equation for the intensity, a parameter that completely describes the radiative field. The intensity is not only a function of 3 spatial coordinates, but also dependent on two directional coordinates and the wavelength. The dependence on so many variables, means that it usually takes a lot of effort to compute the intensity, even if it is a quantity that we are ultimately not interested in. Rather the radiative heat flux, which can be computed from the intensity, is the quantity we need in heat computations.

**Monte Carlo Method,**

This method can be placed in the same class of approaches to numerical radiative heat transfer. It is superficially similar to the Ray Trace Method, but the directions of the rays are chosen randomly rather than through the so-called Discrete Ordinate Method. Furthermore, the method traces energy packets rather than rays. This makes the method suitable for simulation of difficult physical phenomena, but even more expensive than the Ray Tracing Method.

Example:  

Source: Internet