

# Material Properties

Mean Working Temperature:

$$T_M := 25 \text{ }^\circ\text{C}$$

Viscosity:  $\mu_F := 1.975 \cdot 10^{-5} \text{ Pa}\cdot\text{s}$

Mean Absolute Pressure:

$$p_m := 1.01325 \text{ bar}$$

Volume flow:  $Q_F := 0.025 \frac{\text{m}^3}{\text{s}}$

Density:  $\rho_F := \frac{p_m}{287.1 \frac{\text{J}}{\text{kg}\cdot\text{K}} \cdot T_M} = 1.184 \frac{\text{kg}}{\text{m}^3}$

$$\epsilon_r := 0.016 \text{ mm}$$

Surface Roughness

Ref. Diameter:  $D_0 := 20.0 \text{ mm}$   $R_0 := 2.5 \text{ in}$

$$F_0 := \pi \cdot \frac{D_0^2}{4} = 3.142 \times 10^{-4} \text{ m}^2$$

$$V_F := \frac{Q_F}{F_0} = 79.58 \frac{\text{m}}{\text{s}}$$

## Pipe Flow - Smooth and Rough:

$$f_{\epsilon\_D}(\rho, V, D_H, \mu, \epsilon) := \begin{cases} nRe \leftarrow \frac{\rho \cdot V \cdot D_H}{\mu} \\ \Delta_{LIM} \leftarrow \frac{181 \cdot \log(nRe) - 16.4}{nRe} \\ \frac{64.0}{nRe} & \text{if } nRe \leq 2300.0 \\ 0.04 & \text{if } 2300 < nRe \leq 4000 \\ \frac{1}{(1.8 \cdot \log(nRe) - 1.64)^2} & \text{if } 4000 < nRe \wedge \Delta_{LIM} > \frac{\epsilon}{D_H} \\ \frac{0.25}{\left( \log\left( \frac{\epsilon}{D_H^{3.7}} + \frac{5.74}{nRe^{0.9}} \right) \right)^2} & \text{if } 4000 < nRe \end{cases}$$

$$Rey(\rho, V, D, \mu) := \frac{\rho \cdot V \cdot D}{\mu}$$

**Filonenko & Altshul's Formula**

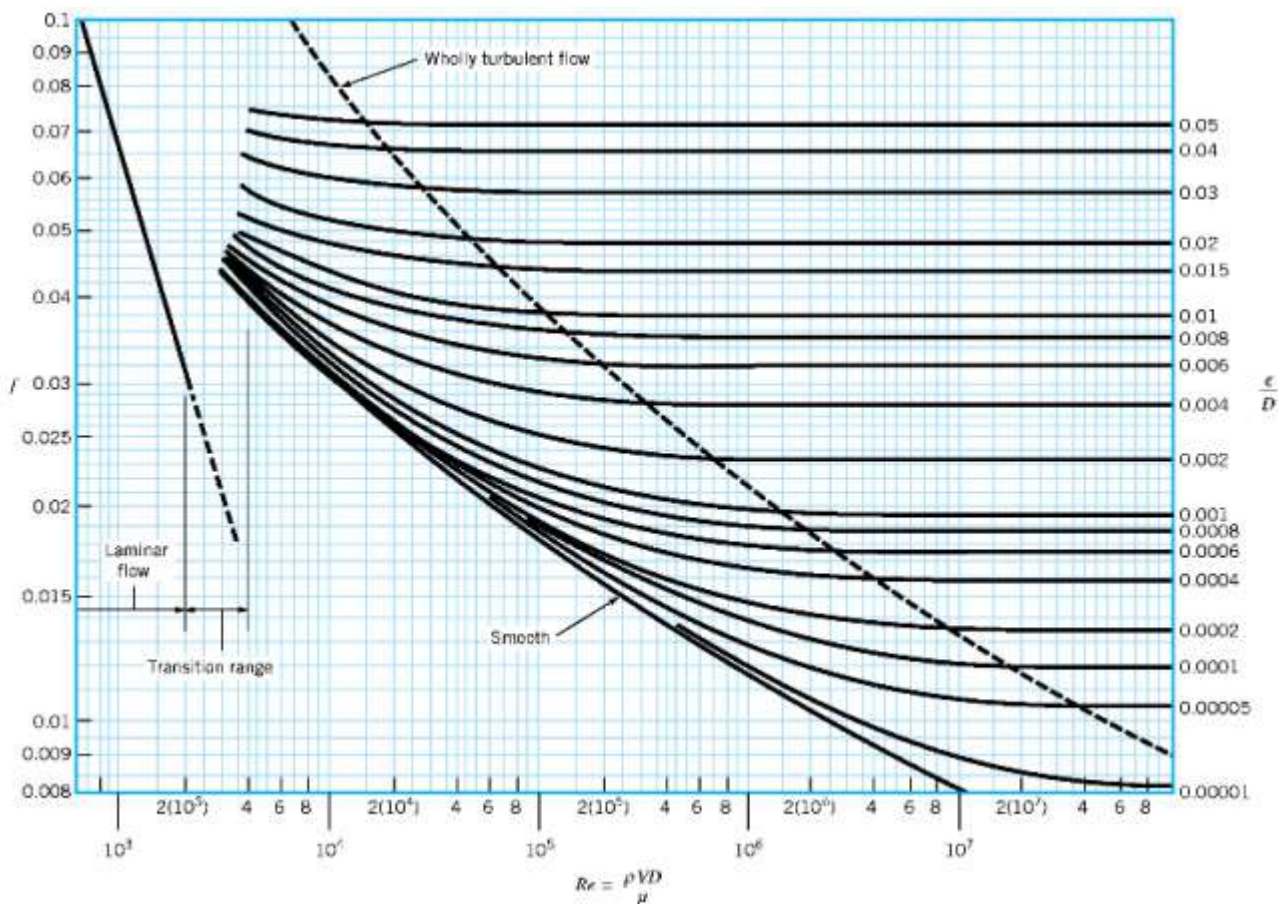
**Colebrook-White-based formula (Roberson & Crowe, 1997)**

$$nRe := Rey(\rho_F, V_F, D_0, \mu_F) = 9.5 \times 10^4$$

Check:  $f_{\epsilon\_D}(\rho_F, V_F, D_0, \mu_F, \epsilon_r) = 0.02173$

$$\frac{\epsilon_r}{D_0} = 0.0008$$

$$Re_{Lm} \leftarrow \frac{217 - 382 \cdot \log\left(\frac{\epsilon_r}{D_0}\right)}{\frac{\epsilon_r}{D_0}} = 1.8 \times 10^4$$



$$f = \frac{1}{\left(2 \cdot \log\left(\frac{2.51}{nRe \cdot \sqrt{f}} + \frac{\epsilon_r}{3.7D_0}\right)\right)^2} \text{ solve, } f \rightarrow 0.02159$$

$$\lambda_x := 0.11 \cdot \left(\frac{\epsilon_r}{D_0} + \frac{68}{nRe}\right)^{0.25} = 0.0217$$

For "stabilized flow" in the region of "purely turbulent flow" and commercial circular tubes with non-uniform roughness of walls

Note: This a transcendental non-linear equation that has to be solved! The Moody chart is just the graphical representation of this formulae). **Accuracy:** +/- 15%

Altshul's Approximation of Colebrook-White Implicit Formula

### Ratio of Average to Centreline Velocity - Duct Flows

$$U(r) = U_c \left(1 - \frac{r}{R}\right)^{\frac{1}{n}} \text{ Here, } U_c \text{ is the centerline velocity}$$

$$V_r(n) := \frac{(n+1) \cdot (2n+1)}{2 \cdot n^2}$$

$$Q = \int_0^R U_c \left(1 - \frac{r}{R}\right)^{\frac{1}{n}} \cdot 2\pi r \cdot dr = 2\pi \cdot U_c \cdot \frac{n^2 \times R^2}{(n+1) \cdot (2n+1)} \text{ where } Q = \text{Volume Flow}$$

$$V_r(7) = 1.224$$

$$Q = \pi R^2 \times \bar{U} \Rightarrow \bar{U} = U_c \cdot \frac{2n^2 \times R^2}{(n+1) \cdot (2n+1)} \Rightarrow U_c = \bar{U} \cdot \frac{(n+1) \cdot (2n+1)}{2n^2}$$

$$V_r(8) = 1.195$$

$$\Rightarrow U(r) = \bar{U} \cdot \frac{(n+1) \cdot (2n+1)}{2n^2} \times \left(1 - \frac{r}{R}\right)^{\frac{1}{n}} = U \cdot V_r \times \left(1 - \frac{r}{R}\right)^{\frac{1}{n}} \text{ where } V_r = \frac{(n+1) \cdot (2n+1)}{2n^2}$$

$$V_r(9) = 1.173$$

$$U_{\text{bar}} := 20 \frac{\text{m}}{\text{s}}$$

$$U_c := U_{\text{bar}} \cdot V_r(7) = 24.5 \frac{\text{m}}{\text{s}}$$

$$V_r(10) = 1.155$$

### Exit from Tube - Circular and Square Tubes

$$\xi_{\text{EXIT}}(\text{Geom}, n) := \begin{cases} \frac{(2 \cdot n + 1)^3 \cdot (n + 1)^3}{4 \cdot n^4 \cdot (2 \cdot n + 3) \cdot (n + 3)} & \text{if } n > 0 \wedge \text{Geom} = \text{"Circ"} \\ \frac{(n + 1)^3}{n^2 \cdot (n + 3)} & \text{if } n > 0 \wedge \text{Geom} = \text{"Plane"} \\ 1.0 & \text{otherwise} \end{cases}$$

In terms of velocity in lower area

Check the Function

$$\xi_{E1} := \xi_{\text{EXIT}}(\text{"Plane"}, 7) = 1.045$$

$$\xi_{E2} := \xi_{\text{EXIT}}(\text{"Circ"}, 7) = 1.058$$

### Sudden Expansion Tube - Circular and Square Tubes

Note: Order of entering cross-section as IS NOT important

$$\xi_{\text{EXP}}(\text{Geom}, A_0, A_2, n) := \begin{cases} M \leftarrow \frac{(2 \cdot n + 1)^2 \cdot (n + 1)}{4 \cdot n^2 \cdot (n + 2)} & \text{if } n > 0 \wedge \text{Geom} = \text{"Circ"} \\ \frac{(n + 1)^2}{n \cdot (n + 2)} & \text{if } n > 0 \wedge \text{Geom} = \text{"Plane"} \end{cases}$$

$$N \leftarrow \xi_{\text{EXIT}}(\text{Geom}, n)$$

Check the Function

$$\left(\frac{\min(A_0, A_2)}{\max(A_0, A_2)}\right)^2 + N - 2M \cdot \frac{\min(A_0, A_2)}{\max(A_0, A_2)} \text{ if } n > 1 \quad \xi_{E3} := \xi_{\text{EXP}}(\text{"Plane"}, 0.5, 1.0, 7) = 0.2$$

$$\xi_{E4} := \xi_{\text{EXP}}(\text{"Circ"}, 0.5, 1.0, 7) = 0.28;$$

$$\left(1 - \frac{\min(A_0, A_2)}{\max(A_0, A_2)}\right)^2 \text{ otherwise}$$

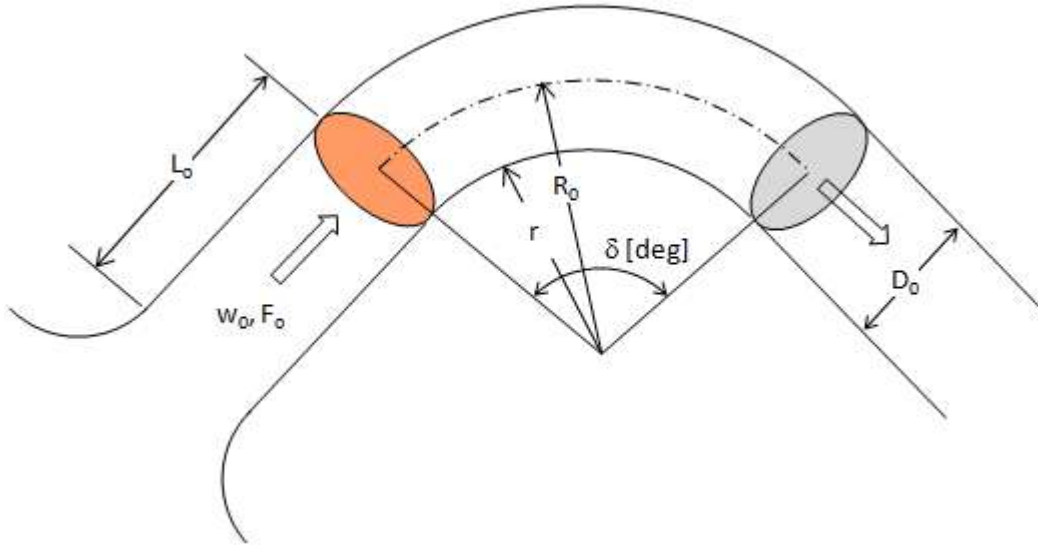
$$\xi_{E5} := \xi_{\text{EXP}}(\text{"Circ"}, 0.5, 1.0, 0) = 0.25$$

### Sudden Contraction

$$\xi_{\text{CONT}}(A_0, A_1) := 0.50 \cdot \left(1.0 - \frac{\min(A_0, A_1)}{\max(A_0, A_1)}\right)^{0.75}$$

Check: User has freedom to enter areas in any order

$$\xi_{\text{CONT}}(0.5, 1.0) = 0.297 \quad \xi_{\text{CONT}}(1.0, 0.5) = 0.297$$



$$\xi_{B.Loc}(R_0, D_0, \delta) := \left| \begin{array}{l} A_1 \leftarrow \begin{cases} (0.9 \cdot \sin(\delta)) & \text{if } \delta \leq 70\text{deg} \\ 1.0 & \text{if } 70\text{deg} < \delta < 100\text{deg} \\ \left(0.7 + 0.35 \cdot \frac{\delta}{90\text{deg}}\right) & \text{otherwise} \end{cases} \\ B_1 \leftarrow \begin{cases} 0.21 \cdot \left(\frac{R_0}{D_0}\right)^{-0.5} & \text{if } \frac{R_0}{D_0} > 1.0 \\ 0.21 \cdot \left(\frac{R_0}{D_0}\right)^{-2.5} & \text{otherwise} \end{cases} \\ A_1 \cdot B_1 \cdot 1.0 \end{array} \right.$$

Note: The order in which  $R_0$  and  $D_0$  are entered is important

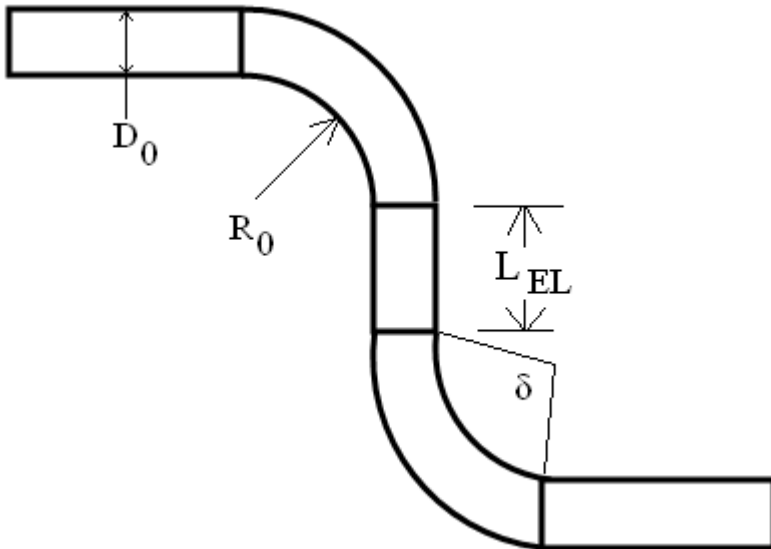
$$\xi_{BND}(R_0, D_0, \delta, \rho, V, \mu) := \left| \begin{array}{l} \xi_1 \leftarrow \xi_{B.Loc}(R_0, D_0, \delta) \\ \lambda \leftarrow f_{\varepsilon\_D}(\rho, V, D_0, \mu, 0.0) \\ \xi_1 + \frac{\pi \cdot \delta}{180\text{deg}} \cdot R_0 \cdot \frac{\lambda}{D_0} \end{array} \right.$$

Walls assumed to be hydraulically smooth:  $\varepsilon = 0$

U-Shaped Joined Bends and Turns in one Plane - Idelchik 3rd Ed, Page 385

$$\xi_{U.BND}(R_0, D_0, \delta, L_{EL}, A, \rho, V, \mu) := \left| \begin{array}{l} \xi_1 \leftarrow \xi_{B.Loc}(R_0, D_0, \delta) \\ \lambda \leftarrow f_{\varepsilon\_D}(\rho, V, D_0, \mu, 0.0) \\ A \cdot \xi_1 + \frac{\pi \cdot \delta}{180\text{deg}} \cdot R_0 \cdot \frac{\lambda}{D_0} \end{array} \right.$$

Specify 'A' based on table on page 385



$$C_{\delta} := \begin{pmatrix} 2.032E-01 & 2.136E-01 & -7.422E-03 & 1.501E-03 & -5.347E-04 & 4.586E-05 & -1.186E-06 \\ 3.950E-01 & 2.247E-01 & 5.838E-02 & -2.680E-02 & 3.999E-03 & -1.821E-04 & 3.599E-06 \\ 6.042E-01 & 6.302E-01 & -2.216E-01 & 3.457E-02 & -2.779E-03 & 1.148E-04 & -1.928E-06 \\ 1.509E+00 & 1.905E-01 & -1.830E-01 & 4.687E-02 & -5.230E-03 & 2.715E-04 & -5.359E-06 \\ 1.052E+00 & 5.668E-01 & -2.999E-01 & 5.982E-02 & -5.624E-03 & 2.548E-04 & -4.489E-06 \\ 1.793E+00 & -3.096E-01 & 1.261E-01 & -2.159E-02 & 1.905E-03 & -8.419E-05 & 1.469E-06 \\ 1.721E+00 & -8.899E-02 & -5.492E-02 & 2.355E-02 & -3.135E-03 & 1.780E-04 & -3.701E-06 \end{pmatrix} \delta_B := \begin{pmatrix} 15 \\ 30 \\ 45 \\ 60 \\ 75 \\ 90 \\ 120 \end{pmatrix}$$

$$a_{15}(L_{EL}) := \sum_{j=0}^6 \left( C_{\delta_{0,j}} \cdot L_{EL}^j \right)$$

$$a_{30}(L_{EL}) := \sum_{j=0}^6 \left( C_{\delta_{1,j}} \cdot L_{EL}^j \right)$$

$$a_{45}(L_{EL}) := \sum_{j=0}^6 \left( C_{\delta_{2,j}} \cdot L_{EL}^j \right)$$

$$a_{60}(L_{EL}) := \sum_{j=0}^6 \left( C_{\delta_{3,j}} \cdot L_{EL}^j \right)$$

$$a_{75}(L_{EL}) := \sum_{j=0}^6 \left( C_{\delta_{4,j}} \cdot L_{EL}^j \right)$$

$$a_{90}(L_{EL}) := \sum_{j=0}^6 \left( C_{\delta_{5,j}} \cdot L_{EL}^j \right)$$

$$a_{120}(L_{EL}) := \sum_{j=0}^6 \left( C_{\delta_{7,j}} \cdot L_{EL}^j \right)$$

**Check:**

Fuction ->	$a_{15}(0.0) = 0.203$	$a_{30}(1.0) = 0.655$	$a_{45}(2.0) = 1.214$	$a_{75}(3.0) = 1.272$	$a_{90}(3.0) = 1.551$
Idelchik Pg 382 ->	0.20	0.65	1.20	1.30	1.37
$\Delta\%$ ->	0%	0%	1.17%	-2.2%	13%

$$A_{S\_BEND}(\delta, L_{EL}) := a_{120} \leftarrow \sum_{j=0}^7 \left( C_{\delta_{6,j}} \cdot L_{EL}^j \right)$$

*(This function has not been used in this calculation)*

$$\xi_{L\_SHARP}(\delta) := \left( 0.95 + \frac{33.5 \text{deg}}{\delta} \right) \cdot \left( 0.95 \cdot \sin\left(\frac{\delta}{2}\right)^2 + 2.05 \cdot \sin\left(\frac{\delta}{2}\right)^4 \right)$$

**Interpolation function for Double Curved Turns (S-Shaped Bends)**

$$I_p(x, x_1, x_2, y, y_1, y_2, z_{11}, z_{12}, z_{21}, z_{22}) := \begin{cases} z_{x1} \leftarrow z_{11} + \frac{x - x_1}{x_2 - x_1} \cdot (z_{21} - z_{11}) \\ z_{x2} \leftarrow z_{12} + \frac{x - x_1}{x_2 - x_1} \cdot (z_{22} - z_{12}) \\ z_{y1} + \frac{y - y_1}{y_2 - y_1} \cdot (z_{x2} - z_{x1}) \end{cases}$$

$$A_S := I_p(22.5, 15.0, 30.0, 2.5, 2.0, 3.0, 0.60, 0.78, 0.88, 1.16) = 0.855 \quad \text{(Sample only)}$$

$$\xi_{sBND}(R_0, D_0, \delta, \rho, V, Dh, \mu, L_{EL}, A_S) := \begin{cases} \lambda \leftarrow f_{\epsilon\_D}(\rho, V, Dh, \mu, 0.0) \\ \xi_{BND}(R_0, D_0, \delta, \rho, V, \mu) \cdot A_S + \frac{\pi \cdot \delta}{180 \text{deg}} \cdot R_0 \cdot \frac{\lambda}{D_0} + \lambda \cdot \frac{L_{EL}}{D_0} \end{cases}$$

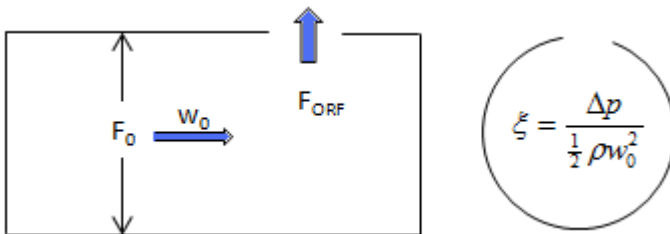
**90 deg Elbow - Made of 3 Elements at the angle 45 deg (Idelchik 3rd Ed, Page 374)**

$$\xi_{EL}(r, D) := \begin{cases} AZ \leftarrow \begin{pmatrix} 1.118112 & -0.6977857 & -0.4818 & 0.70309 & -0.2244795 & -6.968263 \cdot 10^{-4} & 0.010588 & -1.241125 \cdot 10^{-3} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \\ \left[ \sum_{i=0}^7 \left[ AZ_{0,i} \cdot \left( \frac{r}{D} \right)^i \right] \right] \end{cases}$$

**Flow Exit from Side Wall of a Pipe with Recess**

Side (Lateral) Exit from a Duct

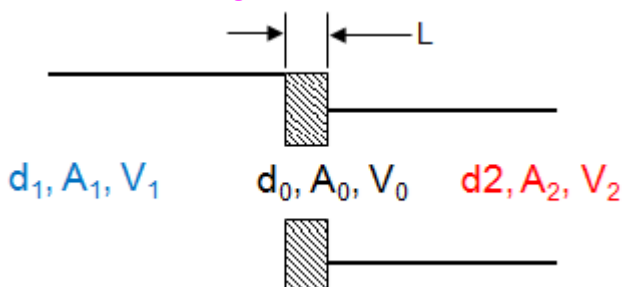
In terms of velocity at **MAIN** cross-section.  
Note: Order of entering cross-section area **IS NOT** important



$$\text{SideExit}(A_{ORF}, A_0) := \begin{cases} Ar \leftarrow \frac{\min(A_{ORF}, A_0)}{\max(A_{ORF}, A_0)} \\ 203.4 - 909.5 \cdot Ar + 1105 \cdot Ar^2 \quad \text{if } Ar \leq 0.40 \\ 107.04 - 440.7 \cdot Ar + 726.8 \cdot Ar^2 - 539.87 \cdot Ar^3 + 149.78 \cdot Ar^4 \quad \text{if } Ar > 0.40 \end{cases}$$

**Orifices - Thick Edge**

In terms of velocity at **ORIFICE** cross-section  
Note: Order of entering cross-section area **IS** important



$$\xi = \frac{\Delta p}{\frac{1}{2} \rho \cdot V_0^2}$$

$$\xi_{\text{ORF}}(A_1, A_0, A_2, D_0, L, \lambda, \text{Type}) := \left| \begin{array}{l} r_L \leftarrow \frac{L}{D_0} \\ \xi_p \leftarrow \begin{cases} (2.4 - r_L) \cdot 10 \cdot \left( 0.25 + \frac{0.535 \cdot r_L^8}{0.05 + r_L^8} \right) & \text{if Type = "Thick"} \\ 0.13 + 0.34 \cdot 10 \cdot \left( 3.45 \cdot r_L + 88.4 \cdot r_L^{2.3} \right) & \text{if Type = "Bevel"} \\ 0.50 & \text{if Type = "Sharp"} \end{cases} \\ \tau_o \leftarrow \begin{cases} \xi_p & \text{if Type = "Thick"} \\ 2 \sqrt{\xi_p} & \text{if Type = "Bevel"} \\ \sqrt{2} & \text{if Type = "Sharp"} \end{cases} \\ \xi_o \leftarrow \begin{cases} 0.50 & \text{if Type = "Thick"} \vee \text{Type = "Sharp"} \\ \xi_p & \text{if Type = "Bevel"} \end{cases} \\ \xi_o \cdot \left( 1 - \frac{A_0}{A_1} \right)^{0.75} + \tau_o \cdot \left( 1 - \frac{A_0}{A_2} \right) \cdot \left( 1 - \frac{A_0}{A_1} \right)^{0.375} + \left( 1 - \frac{A_0}{A_2} \right)^2 + \lambda \cdot \frac{L}{D_0} \end{array} \right.$$

$$\xi_{\text{ORF\_BVL}}(A_1, A_0, A_2, D_0, L, \lambda) := \left| \begin{array}{l} \xi_p \leftarrow 0.13 + 0.34 \cdot 10 \cdot \left[ 3.45 \cdot \frac{L}{D_0} + 88.4 \cdot \left( \frac{L}{D_0} \right)^{2.3} \right] \\ \xi_o \cdot \left( 1 - \frac{A_0}{A_1} \right)^{0.75} + 2 \sqrt{\xi_p} \cdot \left( 1 - \frac{A_0}{A_2} \right) \cdot \left( 1 - \frac{A_0}{A_1} \right)^{0.375} + \left( 1 - \frac{A_0}{A_2} \right)^2 \end{array} \right.$$

$$\xi_{\text{SHARP}}(A_1, A_0, A_2) := \left[ \frac{1}{\sqrt{2}} \cdot \left( 1 - \frac{A_0}{A_1} \right)^{0.375} + \left( 1 - \frac{A_0}{A_2} \right) \right]^2$$

$$\xi_{\text{ORF\_THK}}(A_1, A_0, A_2, D_0, L, \lambda) := \left| \begin{array}{l} r_L \leftarrow \frac{L}{D_0} \\ \tau_o \leftarrow (2.4 - r_L) \cdot 10 \cdot \left( 0.25 + \frac{0.535 \cdot r_L^8}{0.05 + r_L^8} \right) \\ \xi_o \leftarrow 0.50 \\ \xi_o \cdot \left( 1 - \frac{A_0}{A_1} \right)^{0.75} + \tau_o \cdot \left( 1 - \frac{A_0}{A_2} \right) \cdot \left( 1 - \frac{A_0}{A_1} \right)^{0.375} + \left( 1 - \frac{A_0}{A_2} \right)^2 + \lambda \cdot \frac{L}{D_0} \end{array} \right.$$

$$\xi_{\text{EPB}} := \xi_{\text{ORF\_THK}}(49\text{cm}^2, 1.75\text{cm}^2, 49\text{cm}^2, D_H, 3.0\text{mm}, 0.02) = 2.08$$

$$\xi_{\text{EPB2}} := \xi_{\text{ORF}}(49\text{cm}^2, 1.75\text{cm}^2, 49\text{cm}^2, D_H, 3.0\text{mm}, 0.02, \text{"Thick"}) = 2.08$$