Natural Convection Correlations for Various Geometries

Highlights of this MathCAD formulations:

1. Uniform naming convention reduces probability of mistakes (different book have different symbols)

2. All the applicable range of Pr & Ra defined in function leading to foolproof functionality

3. Naming convention with respect of direction of gravity & hence absolute clarity in formulations

4. The lower limits of Pr has been reduced to 0.5 from 0.7 to handle all instances of Air

5. Formula checks by sample calculations after each formulation

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Geometrical Dimension, Material Properties and Operating Conditions

$Pr = 0.70$ $T_S = 100°C$ $T_A = 40°C$

$\beta = \frac{1}{T_A} = 3.193\times 10^{-3}\frac{1}{K}$ $\nu = 18\cdot 10^{-6}\frac{m^2}{s}$ at $[T_A+T_S]/2$ $Lc = 500mm$

$\theta = 0^o$ Vertical Plate $Gr_{Lc} = \frac{g\cdot\beta\cdot(T_S - T_A)\cdot Lc^3}{\nu^2}$ $Ra_L = Gr_{Lc}\cdot Pr$

$\theta = \theta$ Inclined Plate

$\theta = 90^o$ Horizontal Plate *Separate correlation*

$$Regime(Gr_{Lc}) = \begin{cases} \text{"Laminar"} & \text{if } Gr_{Lc} < 10^9 \\ \text{"Turbulent"} & \text{otherwise} \end{cases}$$

$Regime(Gr_{Lc}) = \text{"Laminar"}$

Vertical Plate:

Applicable to V. Cylinder, if: $\frac{D}{L} > \frac{35}{Gr_L^{0.25}}$

L_C = Height of the Plate = L

Restrictions on Pr: **None**

$0^o < \theta < 60^o$: effect of tilt is not significant use $\theta = 0^o$.

$$Nu_{VPx}(Ra_L,\theta) = \begin{cases} 0.59\left(Ra_L\cdot\cos(\theta)\right)^{\frac{1}{4}} & \text{if } 10^4 < Ra_L \le 10^9 \\ 0.14\left(Ra_L\cdot\cos(\theta)\right)^{\frac{1}{3}} & \text{if } 10^9 < Ra_L \le 10^{13} \end{cases}$$

$Nu_{vpx} = Nu_{VPx}(Ra_L, 30°) = 85.4$

$Nu_{vpx1} = Nu_{VPx}(Ra_L, 0°) = 88.6$

$Nu_{vpx2} = Nu_{VPx}(Ra_L, 75°) = 63.2$

Since both Nu and $Ra^{1/3}$ are proportional to L, for $Ra_L > 10^9$, the choice of L is immaterial.

Churchill and Chu (1975)

$$Nu_{VP}(Ra_L,\theta) = \begin{cases} \left[0.825 + \dfrac{0.387\cdot(Ra_L\cdot\cos(\theta))^{\frac{1}{6}}}{\left[1+\left(\dfrac{0.492}{Pr}\right)^{\frac{9}{16}}\right]^{\frac{8}{27}}}\right]^2 & \text{if } 10^9 \le Ra_L \le 10^{12} \\ 0.68 + \dfrac{0.67\cdot(Ra_L\cdot\cos(\theta))^{\frac{1}{4}}}{\left[1+\left(\dfrac{0.492}{Pr}\right)^{\frac{9}{16}}\right]^{\frac{4}{9}}} & \text{if } 0.1 \le Ra_L < 10^9 \end{cases}$$

$$Nu_{VPz} = Nu_{VP}(Ra_L, 75°) = 55.6$$

$$Nu_{VPz1} = Nu_{VP}(Ra_L, 15°) = 77.1$$

$$Nu_{VPz2} = Nu_{VP}(Ra_L, 45°) = 71.3$$

Horizontal Plate:

Upper side of Hot Plate (lower side insulated) / Lower side of Cold Plate (upper side insulated)

Lloyd and Moran (1974)

$$Nu_{HP1}(Ra_L) = \begin{cases} 0.96Ra_L^{\frac{1}{3}} & \text{if } 1 < Ra_L \le 200 \\ 0.59Ra_L^{\frac{1}{4}} & \text{if } 200 < Ra_L \le 2.\cdot 10^4 \\ 0.54Ra_L^{\frac{1}{4}} & \text{if } 2.2\cdot 10^4 < Ra_L \le 8\cdot 10^6 \\ 0.15Ra_L^{\frac{1}{3}} & \text{if } 8\cdot 10^6 < Ra_L \le 10^{11} \end{cases}$$

Lower side of Hot Plate (upper side insulated) / Upper side of Cold Plate (lower side insulated)

Lloyd and Moran (1974)

$$Nu_{HP2}(Ra_L) = \begin{cases} 0.27Ra_L^{\frac{1}{4}} & \text{if } 10^4 \le Ra_L \le 10^{11} \\ \text{"Some variable is out of range!"} & \text{otherwise} \end{cases}$$

Lc = Area / Perimeter

Relative Benchmarking with Vertical Plate:

Width/Length of H. Plate = height of V. Plate

$$Lc_{HP} = \frac{Lc^2}{4\cdot Lc} = 125\cdot mm$$

Pr >= 0.7

$$Nu_{hp1} = Nu_{HP1}\left[Ra_L\cdot\left(\frac{Lc_{HP}}{Lc}\right)^3\right] = 28.7$$

Pr >= 0.7

$$Nu_{hp2} = Nu_{HP2}\left[Ra_L\cdot\left(\frac{Lc_{HP}}{Lc}\right)^3\right] = 14.3$$

HTC from plates hot or cold on both horizontal surfaces is average of the two correlations.

Horizontal Cylinder:

$$Nu_{HC}(Ra_D) = \left[0.60 + \frac{0.387 \cdot Ra_D^{\frac{1}{6}}}{\left[1 + \left(\frac{0.559}{Pr}\right)^{0.5625}\right]^{\frac{8}{27}}}\right]^2$$

Relative Benchmarking with Vertical Plate:

Cylinder with perimeter = Height of V. Plate

$$D_C = \frac{Lc}{\pi} = 0.159\,m$$

$$Nu_{hc} = Nu_{HC}\left[Ra_L \cdot \left(\frac{D_C}{Lc}\right)^3\right] = 32.6$$

$10^{-6} \le Ra_D \le 10^{12}$ All Pr

Sphere:

$$Nu_{SPH}(Ra_D) = \left[2.0 + \frac{0.589 \cdot Ra_D^{0.25}}{\left[1 + \left(\frac{0.469}{Pr}\right)^{0.5625}\right]^{\frac{4}{9}}}\right]$$

Relative Benchmarking:

Sphere with perimeter = Height of V. Plate

$$D_S = \frac{Lc}{\pi} = 0.159\,m$$

$$Nu_{sph} = Nu_{SPH}\left[Ra_L \cdot \left(\frac{D_S}{Lc}\right)^3\right] = 30.9$$

$Ra_D \le 10^{11}$ $Pr \ge 0.7$

Vertical Iso-temperature <u>Parallel Plates</u> separated by distance 'S' - developing & fully developed:

For benchmarking with vertical plates, formulation of Ra-number in terms of L_C is maintained!

$S_v = 500.0mm$ <- Characteristic dimension for Ra

Nu = h.S/k

Bar-Cohen and Rohsenow (1984):

$$Nu_{VIP}(Ra_L) = \left[\frac{576}{\left[Ra_L\left(\frac{S_v}{Lc}\right)^3 \cdot \frac{S_v}{Lc}\right]^2} + \frac{2.83}{\left[Ra_L \cdot \left(\frac{S_v}{Lc}\right)^3 \cdot \frac{S_v}{Lc}\right]^{0.5}}\right]^{-\frac{1}{2}}$$

$$Ra_S = Ra_L \cdot \left(\frac{S_v}{Lc}\right)^3$$

$$Nu_{vip} = Nu_{VIP}(Ra_L) = 89.2$$

Vertical Symmetric Iso-flux <u>Parallel Plates</u> separated by distance 'S':

$$Nu_{VIF}(Ra_L) = \left[\frac{48}{\left[Ra_L \cdot \left(\frac{S_v}{Lc}\right)^3 \cdot \frac{S_v}{Lc}\right]^2} + \frac{2.51}{\left[Ra_L \cdot \left(\frac{S_v}{Lc}\right)^3 \cdot \frac{S_v}{Lc}\right]^{0.5}}\right]^{-\frac{1}{2}}$$

$$Nu_{vif} = Nu_{VIF}(Ra_L) = 94.7$$

Vertical Isothermal-Adibatic Parallel Plates separated by distance 'S':

$$Nu_{IAD}(Ra_L) = \left[\frac{144}{\left[Ra_L \cdot \left(\frac{S_V}{Lc}\right)^3 \cdot \frac{S_V}{Lc}\right]^2} + \frac{2.87}{\left[Ra_L \cdot \left(\frac{S_V}{Lc}\right)^3 \cdot \frac{S_V}{Lc}\right]^{0.5}} \right]^{-\frac{1}{2}}$$

$$Nu_{iad} = Nu_{IAD}(Ra_L) = 88.6$$

Vertical Flux-Adibatic Parallel Plates separated by distance 'S':

$$Nu_{FAD}(Ra_L) = \left[\frac{24}{\left[Ra_L \cdot \left(\frac{S_V}{Lc}\right)^3 \cdot \frac{S_V}{Lc}\right]^2} + \frac{2.51}{\left[Ra_L \cdot \left(\frac{S_V}{Lc}\right)^3 \cdot \frac{S_V}{Lc}\right]^{0.5}} \right]^{-\frac{1}{2}}$$

$$Nu_{fad} = Nu_{FAD}(Ra_L) = 94.7$$

Vertical Rectangular Enclosure, Dimension along gravity L_C, perpendicular to gravity 'S':

$S_V = 500.0mm$ <- Characteristic dimension for Ra ***Heated from sides, also called Tall Enclosures***

Berkovsky & Polevikov for aspect ratio (A.R.) up to 10 and MacGregor & Emery (1969) for A.R. > 10:

$$Nu_{VEN}(Ra_S, S, L) = \begin{cases} 0.18\left(\frac{Pr}{0.2 + Pr} Ra_S\right)^{0.29} & \text{if } \frac{Pr \cdot Ra_S}{0.2 + Pr} \geq 1000 \wedge 1 \leq \frac{L}{S} \leq 2 \wedge 10^{-3} \leq Pr \leq 10^5 \\ 0.22\left(\frac{Pr}{0.2 + Pr} Ra_S\right)^{0.28} \cdot \left(\frac{L}{S}\right)^{\frac{1}{4}} & \text{if } 10^3 \leq Ra_S \leq 10^{13} \wedge 2 < \frac{L}{S} \leq 10 \wedge Pr \leq 10^5 \\ 0.42 Ra_S^{\frac{1}{4}} \cdot Pr^{0.012} \cdot \left(\frac{L}{S}\right)^{\frac{1}{3}} & \text{if } 10^4 < Ra_S \leq 10^7 \wedge 10 < \frac{L}{S} \leq 40 \wedge 0.5 \leq Pr \leq 2 \cdot 10^4 \\ \text{"Variable(s) seems to be out of range!"} & \text{otherwise} \end{cases}$$

$$\frac{Pr \cdot \left[Ra_L \cdot \left(\frac{S_V}{Lc}\right)^3\right]}{0.2 + Pr} = 3.9 \times 10^8 \qquad Ra_L \cdot \left(\frac{S_V}{Lc}\right)^3 = 5.1 \times 10^8 \qquad Nu_{ven} = Nu_{VEN}\left[Ra_L \cdot \left(\frac{S_V}{Lc}\right)^3, S_V, Lc\right] = 56$$

Inclined Rectangular Enclosure, Dimension along plates L_C, perpendicular to plates 'S':

$\theta_H = 50°$ $0° < \theta_H <= 70°$ $\cos(\theta_H) = 0.643$

$$nZ(x) = \begin{cases} x & \text{if } x \geq 0 \\ 0 & \text{if } x < 0 \end{cases}$$

(Hollands et al)

$$\mathrm{Nu_{INC}}\left(\mathrm{Ra_S}, S, L, \theta\right) = \begin{array}{|l} 1 \quad \text{if } \mathrm{Ra_S}\cdot\cos(\theta) \le 1708 \wedge 12 \le \frac{L}{S} \wedge 0° < \theta \le 70° \\ \xi \leftarrow \left(1 - \frac{1708}{\mathrm{Ra_S}\cdot\cos(\theta)}\right) \\ \psi \leftarrow 1 - \frac{1708\cdot(\sin(1.8\cdot\theta))^{1.6}}{\mathrm{Ra_S}\cdot\cos(\theta)} \\ \zeta \leftarrow \frac{\left(\mathrm{Ra_S}\cdot\cos(\theta)\right)^{0.333}}{18} - 1 \\ 1 + 1.44\,\mathrm{nZ}(\xi)\cdot\mathrm{nZ}(\psi) + \mathrm{nZ}(\zeta) \quad \text{if } 1708 < \mathrm{Ra_S} \le 10^5 \wedge \frac{L}{S} \ge 12 \wedge 0° < \theta \le 70° \\ \text{"Variable(s) seems to be out of range!"} \quad \text{otherwise} \end{array}$$

$$\mathrm{Ra_L}\cdot\left(\frac{S_V}{Lc}\right)^3 = 5.1\times10^8 \qquad \mathrm{Nu_{inc}} = \mathrm{Nu_{INC}}\left[\mathrm{Ra_L}\cdot\left(\frac{S_V}{Lc}\right)^3, S_V, Lc, \theta_H\right] = \text{"Variable(s) seems to be out of range!"}$$

Critical tilt angle after which the boundary layer will become thermally unstable.

Aspect Ratio = $(1.0\ \ 3.0\ \ 6.0\ \ 12\ \ \theta > 12)$

Critical Tile Angle = $(155\ \ 127\ \ 120\ \ 113\ \ 110)$

(Arnold et al - 1976)

Inclined SQUARE Enclosure, tilt-angle with respect of horizontal direction:

For horizontal square enclosure, the heat transfer is solely by conduction and hence Nu(0º) = 1

Horizontal Rectangular Enclosure, Dimension along gravity L_C, perpendicular to gravity 'S':

Heated from Bottom, ***expression for Nu applicable to Top surface***. Nu for bottom surface to be estimatec from applied heat flux or indirectly from heat loss throught the top surface. The aspect ratio "vertical height"/ "horizontal length" must be sufficiently high (~ 10 or more) to reduce effect of side-walls.

Note that the designation of geometrical parameter is same as vertical enclosure and hence refer to different "physically significant dimensions".

Jacob: Also called Shallow Enclosures

$$Nu_{JCB}(Ra_L) = \begin{cases} 0.195 Ra_L^{\frac{1}{4}} & \text{if } 10^4 < Ra_L \le 4\cdot 10^5 \wedge 0.5 \le Pr \le 2.0 \\ 0.068 Ra_L^{\frac{1}{3}} & \text{if } 4\cdot 10^5 < Ra_L \le 10^9 \\ \text{"Variable(s) seems to be out of range!"} & \text{otherwise} \end{cases}$$

$$Nu_{JCB}(Ra_L) = 54.2$$

Globe and Dropkin (1959):

$$Nu_{GLD}(Ra_L) = \begin{cases} 0.069 Ra_L^{\frac{1}{3}} \cdot Pr^{0.074} & \text{if } 3\cdot 10^5 < Ra_L \le 7\cdot 10^9 \\ \text{"Variable(s) seems to be out of range"} & \text{otherwise} \end{cases}$$

$$Nu_{GLD}(Ra_L) = 53.6$$

Concentric Cylinders and Spheres: **$L_C = (D_o - D_i) / 2$**

Raithby and Hollands (1975):

$$Q_{ANN}(T_i, T_o, D_i, D_o, k, Ra) = \left| \begin{array}{l} F \leftarrow \dfrac{\left(\ln\left(\dfrac{D_o}{D_i}\right)\right)^4}{\left[\dfrac{(D_o - D_i)}{2}\right]^3 \cdot \left(\dfrac{1}{D_o^{0.6}} + \dfrac{1}{D_i^{0.6}}\right)^5} \\ k_E \leftarrow \left(\dfrac{Pr}{0.861 + Pr}\right)^{\frac{1}{4}} \cdot (Ra \cdot F)^{\frac{1}{4}} \\ \left[\dfrac{2\cdot\pi\cdot(0.386 k \cdot k_E)}{\ln\left(\dfrac{D_o}{D_i}\right)} \cdot (T_i - T_o)\right] \text{ if } 0.5 \le Pr \le 6\cdot 10^3 \wedge 10^2 \le F\cdot Ra \le 10^7 \\ \text{"Variable(s) seems to be out of range"} \quad \text{otherwise} \end{array} \right.$$

For $F*Ra_L < 100$, natural convection currents are negligible and the annular passage can be assumed to be static fluid that is $k_{EF}F = k$. Also note that 2 Nu will need to be defined for inner and outer diameters.

$T_i = T_S \qquad T_o = T_A \qquad k = 0.026 \frac{W}{m \cdot K} \qquad D_i = 0.1m \qquad D_o = 0.4m$

Thermo-physical properties at $(T_i+T_o)/2$.

$$D_H = \frac{D_o - D_i}{2} = 150 \cdot mm$$

$$F_{ann} = \frac{\left(\ln\left(\frac{D_o}{D_i}\right)\right)^4}{\left[\frac{(D_o - D_i)}{2}\right]^3 \cdot \left(\frac{1}{D_o^{0.6}} + \frac{1}{D_i^{0.6}}\right)^5} = 0.18$$

$$Ra_{ann} = F_{ann} \cdot \left[Ra_L \cdot \left(\frac{D_o - D_i}{2 \cdot Lc}\right)^3\right] = 2.5 \times 10^6$$

$$Q_{ann} = Q_{ANN}(T_i, T_o, D_i, D_o, k, Ra_{ann}) = 57.6 \cdot \frac{W}{m}$$

$$Ra_{Di} = Ra_L \cdot \left(\frac{D_i}{Lc}\right)^3 = 4.1 \times 10^6$$

Alternate formulation based on inner diameter of the annulus is:

$$q_{ANN}(T_i, T_o, D_i, D_o, k, Ra_{Di}) = \begin{cases} \dfrac{2.425 \cdot k \cdot (T_i - T_o)}{\left[1 + \left(\frac{D_i}{D_o}\right)^{0.6}\right]^{1.25}} \cdot \left(\dfrac{Pr \cdot Ra_{Di}}{0.861 + Pr}\right)^{\frac{1}{4}} & \text{if } 0.6 \le Pr \le 6000 \wedge Ra_{Di} \le 10^7 \\ \text{"Variable(s) seems to be out of range"} & \text{otherwise} \end{cases}$$

$$q_{ann} = q_{ANN}(T_i, T_o, D_i, D_o, k, Ra_{Di}) = 88.5 \cdot \frac{W}{m}$$

Alternate formulation based on inner diameter of the **concentric spheres** is:

$$q_{ASP}(T_i, T_o, D_i, D_o, k, Ra_{Di}) = \begin{cases} \dfrac{2.325 \cdot k \cdot (T_i - T_o)}{\left[1 + \left(\frac{D_i}{D_o}\right)^{1.4}\right]^{1.25}} \cdot \left(\dfrac{Pr \cdot Ra_{Di}}{0.861 + Pr}\right)^{\frac{1}{4}} & \text{if } 0.6 \le Pr \le 4000 \wedge Ra_{Di} \le 10^4 \\ \text{"Variable(s) seems to be out of range"} & \text{otherwise} \end{cases}$$

$$q_{asp} = q_{ASP}(T_i, T_o, D_i, D_o, k, Ra_{Di}) = \text{"Variable(s) seems to be out of range"} \cdot \frac{W}{m}$$

Optimum fin spacing for a vertical heat sink

Rohsenow and Cohen: $Sv_{OPT}(L_c) = 2.714 \cdot \frac{L_c}{Ra_L^{0.25}}$

L_C is the characteristic lenth for Ra which is vertical height of the fins

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Square Open Cavity: (a) All 3 walls isothermal (b) Wall facing opening isothermal, other adiabatic (c) partially open cavity

Applications are: (i) Solar thermal energy receivers (ii) Electronin Cooling Chambers

(In preparation)

Other correlations in Natual Convective with effects of Variable Properties & B.C.

Siebers et al:

$$Nu_{VPQ}(Gr_L) = \begin{cases} 0.404\,(Gr_L)^{\frac{1}{4}} \cdot \left(\frac{T_A}{T_S}\right)^{0.04} & \text{if } Gr_L \le 10^9 \\ 0.098\,(Gr_L)^{\frac{1}{3}} \cdot \left(\frac{T_A}{T_S}\right)^{0.14} & \text{if } 10^9 < Ra_L \end{cases}$$

Plate surface at Uniform Heat Flux

$$Nu_{VPT}(Gr_L) = \begin{cases} 0.356\,(Gr_L)^{\frac{1}{4}} \cdot \left(\frac{T_A}{T_S}\right)^{0.04} & \text{if } Gr_L \le 10^9 \\ 0.098\,(Gr_L)^{\frac{1}{3}} \cdot \left(\frac{T_A}{T_S}\right)^{0.14} & \text{if } 10^9 < Ra_L \end{cases}$$

Plate surface at Uniform Temperature

Symmetrically heated isothermal plates (Elenbass, 1942)

$$Nu_{PPT}(Ra_S, L, S) = \frac{Ra_S}{24} \cdot \frac{S}{L} \cdot \left(1 - e^{\frac{-35}{Ra_S \cdot \frac{S}{L}}}\right)^{0.75}$$

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