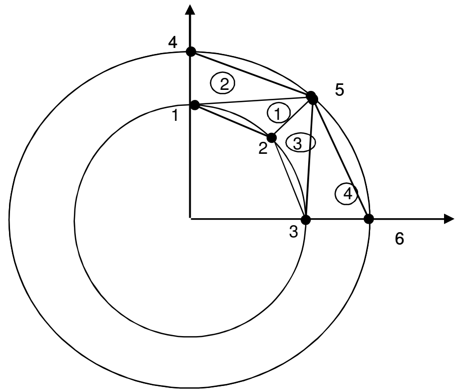
Geometry



Material

Properties:

Mesh

The "Element Definition Vector" determines the order in which the elements displacements are stored in the vector.

Stiffness Matrix

Material Properties:

E [Pa]	1.00E+11
ν [—]	0.25
t [m]	0.001
λ	4.00E+10
=Lame's Constant	
λ_1	1.07E+11
	$=E/(1 - \nu^2)$

Element Definition vector

Element No.	Node Number		
1	1	2	5
2	1	5	4
3	2	3	5
4	3	6	5

Node No.	**X**	**Y**	**Z**
1	0	0.2	0
2	0.14142	0.14142	0
3	0.2	0	0
4	0	0.3	0
5	0.21213	0.21213	0
6	0.3	0	0

Elasticity or Element Stiffness or Stress-Strain Matrix: D

$$= \lambda_1 \ast \begin{pmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & (1-\nu)/2 \end{pmatrix}$$

$$\mathbf{D}= \begin{pmatrix} 1.07E+11 & 2.67E+10 & 0 \\ 2.67E+10 & 1.07E+11 & 0 \\ 0 & 0 & 4.0E+10 \end{pmatrix}$$

$C_1 =$ 1.07E+11

$C_2 =$ 0.25 $C_1 . C_2 =$ 2.67E+10

$C_{12} =$ 4.00E+10

Gradient Matrix:

$$\Delta = 0.5*[(X_2-X_1).(Y_3-Y_1) - (X_3-X_1).(Y_2-Y_1)]$$

$$\begin{pmatrix} a_i & a_j & a_m \\ b_i & b_j & b_m \\ c_i & c_j & c_m \end{pmatrix} \qquad =1/2\Delta . \begin{pmatrix} b_i & 0 & b_j & 0 & b_m & 0 \\ 0 & c_i & 0 & c_j & 0 & c_m \\ c_i & b_i & c_j & b_j & c_m & b_m \end{pmatrix}$$

Gradient Matrix based on element Shape Function: B

$$[A_1].2\Delta= \begin{array}{c|ccc} & 1 & 2 & 5 \\ \hline 1 & x_2y_5 - x_5y_2 & x_5y_1 - x_1y_5 & x_1y_2 - x_2y \\ 2 & y_2 - y_5 & y_5 - y_1 & y_1 - y_2 \\ 5 & x_5 - x_2 & x_1 - x_5 & x_2 - x_1 \end{array} = \begin{pmatrix} a^1_{11} & a^1_{12} & a^1_{13} \\ b^1_{11} & b^1_{12} & b^1_{13} \\ c^1_{11} & c^1_{12} & c^1_{13} \end{pmatrix} = \overset{A}{\begin{pmatrix} 0.000 & 0.042 & -0.028 \\ -0.071 & 0.012 & 0.059 \\ 0.071 & -0.212 & 0.141 \end{pmatrix}} \quad \overset{\Delta}{0.0071} \quad \overset{B}{\begin{pmatrix} -5.000E+0 & 0.000E+0 & 8.577E-1 & 0.000E+0 & 4.142E+0 & 0.000E+0 \\ 0.000E+0 & 5.000E+0 & 0.000E+0 & -1.500E+1 & 0.000E+0 & 1.000E+1 \\ 5.000E+0 & -5.000E+0 & -1.500E+1 & 8.577E-1 & 1.000E+1 & 4.142E+0 \end{pmatrix}}$$

$$[A_2].2\Delta= \begin{array}{c|ccc} & 1 & 5 & 4 \\ \hline 1 & x_5y_4 - x_4y_5 & x_4y_1 - x_1y_4 & x_1y_5 - x_5y \\ 5 & y_5 - y_4 & y_4 - y_1 & y_1 - y_5 \\ 4 & x_4 - x_5 & x_1 - x_4 & x_5 - x_1 \end{array} = \begin{pmatrix} a^2_{11} & a^2_{12} & a^2_{13} \\ b^2_{11} & b^2_{12} & b^2_{13} \\ c^2_{11} & c^2_{12} & c^2_{13} \end{pmatrix} = \begin{pmatrix} 0.064 & 0.000 & -0.042 \\ -0.088 & 0.100 & -0.012 \\ -0.212 & 0.000 & 0.212 \end{pmatrix} \quad 0.0106 \quad \begin{pmatrix} -4.142E+0 & 0.000E+0 & 4.714E+0 & 0.000E+0 & -5.718E-1 & 0.000E+0 \\ 0.000E+0 & -1.000E+1 & 0.000E+0 & 0.000E+0 & 0.000E+0 & 1.000E+1 \\ -1.000E+1 & -4.142E+0 & 0.000E+0 & 4.714E+0 & 1.000E+1 & -5.718E-1 \end{pmatrix}$$

$$[A_3].2\Delta= \begin{array}{c|ccc} & 2 & 3 & 5 \\ \hline 2 & x_3y_5 - x_5y_3 & x_5y_2 - x_2y_5 & x_2y_3 - x_3y_2 \\ 3 & y_3 - y_5 & y_5 - y_2 & y_2 - y_3 \\ 5 & x_5 - x_3 & x_2 - x_5 & x_3 - x_2 \end{array} = \begin{pmatrix} a^3_{11} & a^3_{12} & a^3_{13} \\ b^3_{11} & b^3_{12} & b^3_{13} \\ c^3_{11} & c^3_{12} & c^3_{13} \end{pmatrix} = \begin{pmatrix} 0.042 & 0.000 & -0.028 \\ -0.212 & 0.071 & 0.141 \\ 0.012 & -0.071 & 0.059 \end{pmatrix} \quad 0.0071 \quad \begin{pmatrix} -1.500E+1 & 0.000E+0 & 5.000E+0 & 0.000E+0 & 1.000E+1 & 0.000E+0 \\ 0.000E+0 & 8.577E-1 & 0.000E+0 & -5.000E+0 & 0.000E+0 & 4.142E+0 \\ 8.577E-1 & -1.500E+1 & -5.000E+0 & 5.000E+0 & 4.142E+0 & 1.000E+1 \end{pmatrix}$$

$$[A_4].2\Delta= \begin{array}{c|ccc} & 3 & 6 & 5 \\ \hline 3 & x_6y_5 - x_5y_6 & x_5y_3 - x_3y_5 & x_3y_6 - x_6y_3 \\ 6 & y_6 - y_5 & y_5 - y_3 & y_3 - y_6 \\ 5 & x_5 - x_6 & x_3 - x_5 & x_6 - x_3 \end{array} = \begin{pmatrix} a^4_{11} & a^4_{12} & a^4_{13} \\ b^4_{11} & b^4_{12} & b^4_{13} \\ c^4_{11} & c^4_{12} & c^4_{13} \end{pmatrix} = \begin{pmatrix} 0.064 & -0.042 & 0.000 \\ -0.212 & 0.212 & 0.000 \\ -0.088 & -0.012 & 0.100 \end{pmatrix} \quad 0.0106 \quad \begin{pmatrix} -1.000E+1 & 0.000E+0 & 1.000E+1 & 0.000E+0 & 0.000E+0 & 0.000E+0 \\ 0.000E+0 & -4.142E+0 & 0.000E+0 & -5.718E-1 & 0.000E+0 & 4.714E+0 \\ -4.142E+0 & -1.000E+1 & -5.718E-1 & 1.000E+1 & 4.714E+0 & 0.000E+0 \end{pmatrix}$$

Element Stiffness Matrix: k

$$[B_1^T]= \begin{pmatrix} -5.000E+00 & 0.000E+00 & 5.000E+00 \\ 0.000E+00 & 5.000E+00 & -5.000E+00 \\ 8.577E-01 & 0.000E+00 & -1.500E+01 \\ 0.000E+00 & -1.500E+01 & 8.577E-01 \\ 4.142E+00 & 0.000E+00 & 1.000E+01 \\ 0.000E+00 & 1.000E+01 & 4.142E+00 \end{pmatrix} \qquad \overset{\mathbf{[B_1^T] . [D]}}{\begin{pmatrix} -5.3E+11 & -1.3E+11 & 2.0E+11 \\ 1.3E+11 & 5.3E+11 & -2.0E+11 \\ 9.1E+10 & 2.3E+10 & -6.0E+11 \\ -4.0E+11 & -1.6E+12 & 3.4E+10 \\ 4.4E+11 & 1.1E+11 & 4.0E+11 \\ 2.7E+11 & 1.1E+12 & 1.7E+11 \end{pmatrix}} \qquad 1$$

$$[k] = \quad \mathbf{t.\ \Delta.\ [B_1^T] . [D] . [B]}$$

$$\begin{array}{c|ccc} & \mathbf{1} & \mathbf{2} & \mathbf{5} \\ \hline \mathbf{1} & \begin{pmatrix} 2.59E+7 & -1.18E+7 \\ -1.18E+7 & 2.59E+7 \end{pmatrix} & \begin{pmatrix} -2.44E+7 & 1.54E+7 \\ 2.20E+7 & -5.78E+7 \end{pmatrix} & \begin{pmatrix} -1.48E+6 & -3.57E+6 \\ -1.02E+7 & 3.19E+7 \end{pmatrix} \\ \mathbf{2} & \begin{pmatrix} -2.44E+7 & 2.20E+7 \\ 1.54E+7 & -5.78E+7 \end{pmatrix} & \begin{pmatrix} 6.42E+7 & -6.07E+6 \\ -6.07E+6 & 1.70E+8 \end{pmatrix} & \begin{pmatrix} -3.97E+7 & -1.60E+7 \\ -9.29E+6 & -1.12E+8 \end{pmatrix} \\ \mathbf{5} & \begin{pmatrix} -1.48E+6 & -1.02E+7 \\ -3.57E+6 & 3.19E+7 \end{pmatrix} & \begin{pmatrix} -3.97E+7 & -9.29E+6 \\ -1.60E+7 & -1.12E+8 \end{pmatrix} & \begin{pmatrix} 4.12E+7 & 1.95E+7 \\ 1.95E+7 & 8.03E+7 \end{pmatrix} \end{array} = \begin{pmatrix} [k^1_{11}] & [k^1_{12}] & [k^1_{15}] \\ [k^1_{21}] & [k^1_{22}] & [k^1_{25}] \\ [k^1_{51}] & [k^1_{52}] & [k^1_{55}] \end{pmatrix}$$

15-10-2009

Element Stiffness Matrix: k

$$[B_2^T]= \begin{pmatrix} -4.142E+00 & 0.000E+00 & -1.000E+01 \\ 0.000E+00 & -1.000E+01 & -4.142E+00 \\ 4.714E+00 & 0.000E+00 & 0.000E+00 \\ 0.000E+00 & 0.000E+00 & 4.714E+00 \\ -5.718E-01 & 0.000E+00 & 1.000E+01 \\ 0.000E+00 & 1.000E+01 & -5.718E-01 \end{pmatrix} \quad \begin{pmatrix} -4.4E+11 & -1.1E+11 & -4.0E+11 \\ -2.7E+11 & -1.1E+12 & -1.7E+11 \\ 5.0E+11 & 1.3E+11 & 0.0E+00 \\ 0.0E+00 & 0.0E+00 & 1.9E+11 \\ -6.1E+10 & -1.5E+10 & 4.0E+11 \\ 2.7E+11 & 1.1E+12 & -2.3E+10 \end{pmatrix}$$

$$2 \quad \begin{array}{c|cc|cc|cc} & 1 & & 5 & & 4 & \\ \hline 1 & 6.18E+7 & 2.93E+7 & -2.21E+7 & -2.00E+7 & -3.97E+7 & -9.29E+6 \\ & 2.93E+7 & 1.20E+8 & -1.33E+7 & -8.28E+6 & -1.60E+7 & -1.12E+8 \\ \hline 5 & -2.21E+7 & -1.33E+7 & 2.51E+7 & 0.00E+0 & -3.05E+6 & 1.33E+7 \\ & -2.00E+7 & -8.28E+6 & 0.00E+0 & 9.43E+6 & 2.00E+7 & -1.14E+6 \\ \hline 4 & -3.97E+7 & -1.60E+7 & -3.05E+6 & 2.00E+7 & 4.28E+7 & -4.04E+6 \\ & -9.29E+6 & -1.12E+8 & 1.33E+7 & -1.14E+6 & -4.04E+6 & 1.13E+8 \end{array} = \begin{pmatrix} [k^2_{11}] & [k^2_{12}] & [k^2_{13}] \\ [k^2_{21}] & [k^2_{22}] & [k^2_{23}] \\ [k^2_{31}] & [k^2_{32}] & [k^2_{33}] \end{pmatrix}$$

$$[B_3^T]= \begin{pmatrix} -1.500E+01 & 0.000E+00 & 8.577E-01 \\ 0.000E+00 & 8.577E-01 & -1.500E+01 \\ 5.000E+00 & 0.000E+00 & -5.000E+00 \\ 0.000E+00 & -5.000E+00 & 5.000E+00 \\ 1.000E+01 & 0.000E+00 & 4.142E+00 \\ 0.000E+00 & 4.142E+00 & 1.000E+01 \end{pmatrix} \quad \begin{pmatrix} -1.6E+12 & -4.0E+11 & 3.4E+10 \\ 2.3E+10 & 9.1E+10 & -6.0E+11 \\ 5.3E+11 & 1.3E+11 & -2.0E+11 \\ -1.3E+11 & -5.3E+11 & 2.0E+11 \\ 1.1E+12 & 2.7E+11 & 1.7E+11 \\ 1.1E+11 & 4.4E+11 & 4.0E+11 \end{pmatrix}$$

$$3 \quad \begin{array}{c|cc|cc|cc} & 2 & & 3 & & 5 & \\ \hline 2 & 1.70E+8 & -6.07E+6 & -5.78E+7 & 1.54E+7 & -1.12E+8 & -9.29E+6 \\ & -6.07E+6 & 6.42E+7 & 2.20E+7 & -2.44E+7 & -1.60E+7 & -3.97E+7 \\ \hline 3 & -5.78E+7 & 2.20E+7 & 2.59E+7 & -1.18E+7 & 3.19E+7 & -1.02E+7 \\ & 1.54E+7 & -2.44E+7 & -1.18E+7 & 2.59E+7 & -3.57E+6 & -1.48E+6 \\ \hline 5 & -1.12E+8 & -1.60E+7 & 3.19E+7 & -3.57E+6 & 8.03E+7 & 1.95E+7 \\ & -9.29E+6 & -3.97E+7 & -1.02E+7 & -1.48E+6 & 1.95E+7 & 4.12E+7 \end{array} = \begin{pmatrix} [k^3_{11}] & [k^3_{12}] & [k^3_{13}] \\ [k^3_{21}] & [k^3_{22}] & [k^3_{23}] \\ [k^3_{31}] & [k^3_{32}] & [k^3_{33}] \end{pmatrix}$$

$$[B_4^T]= \begin{pmatrix} -1.000E+01 & 0.000E+00 & -4.142E+00 \\ 0.000E+00 & -4.142E+00 & -1.000E+01 \\ 1.000E+01 & 0.000E+00 & -5.718E-01 \\ 0.000E+00 & -5.718E-01 & 1.000E+01 \\ 0.000E+00 & 0.000E+00 & 4.714E+00 \\ 0.000E+00 & 4.714E+00 & 0.000E+00 \end{pmatrix} \quad \begin{pmatrix} -1.1E+12 & -2.7E+11 & -1.7E+11 \\ -1.1E+11 & -4.4E+11 & -4.0E+11 \\ 1.1E+12 & 2.7E+11 & -2.3E+10 \\ -1.5E+10 & -6.1E+10 & 4.0E+11 \\ 0.0E+00 & 0.0E+00 & 1.9E+11 \\ 1.3E+11 & 5.0E+11 & 0.0E+00 \end{pmatrix}$$

$$4 \quad \begin{array}{c|cc|cc|cc} & 3 & & 6 & & 5 & \\ \hline 3 & 1.20E+8 & 2.93E+7 & -1.12E+8 & -1.60E+7 & -8.28E+6 & -1.33E+7 \\ & 2.93E+7 & 6.18E+7 & -9.29E+6 & -3.97E+7 & -2.00E+7 & -2.21E+7 \\ \hline 6 & -1.12E+8 & -9.29E+6 & 1.13E+8 & -4.04E+6 & -1.14E+6 & 1.33E+7 \\ & -1.60E+7 & -3.97E+7 & -4.04E+6 & 4.28E+7 & 2.00E+7 & -3.05E+6 \\ \hline 5 & -8.28E+6 & -2.00E+7 & -1.14E+6 & 2.00E+7 & 9.43E+6 & 0.00E+0 \\ & -1.33E+7 & -2.21E+7 & 1.33E+7 & -3.05E+6 & 0.00E+0 & 2.51E+7 \end{array} = \begin{pmatrix} [k^4_{11}] & [k^4_{12}] & [k^4_{13}] \\ [k^4_{21}] & [k^4_{22}] & [k^4_{23}] \\ [k^4_{31}] & [k^4_{32}] & [k^4_{33}] \end{pmatrix}$$

STRUCTURAL (GLOBAL) STIFFNESS MATRIX:

$$\begin{array}{c|cccccc} & 1 & 2 & 3 & 4 & 5 & 6 \\ \hline 1 & [k^1_{11}] + [k^2_{11}] & [k^1_{12}] & & [k^2_{13}] & [k^1_{13}] + [k^2_{12}] & \\ 2 & [k^1_{21}] & [k^1_{22}] + [k^3_{11}] & [k^3_{12}] & & [k^1_{23}] + [k^3_{13}] & \\ 3 & & [k^3_{21}] & [k^3_{22}] + [k^4_{11}] & & [k^3_{23}] + [k^4_{13}] & [k^4_{12}] \\ 4 & [k^2_{31}] & & & [k^2_{33}] & [k^2_{32}] & \\ 5 & [k^1_{31}] + [k^2_{21}] & [k^1_{32}] + [k^3_{31}] & [k^3_{32}] + [k^4_{31}] & [k^2_{23}] & [k^1_{33}]+[k^4_{33}]+ [k^2_{22}]+[k^3_{33}] & [k^4_{32}] \\ 6 & & & [k^4_{21}] & & [k^4_{23}] & [k^4_{22}] \end{array} \quad \begin{matrix} u_1 \\ v_1 \\ u_2 \\ v_2 \\ u_3 \\ v_3 \\ u_4 \\ v_4 \\ u_5 \\ v_5 \\ u_6 \\ v_6 \end{matrix} = \begin{matrix} X_1 \\ Y_1 \\ X_2 \\ Y_2 \\ X_3 \\ Y_3 \\ X_4 \\ Y_4 \\ X_5 \\ Y_5 \\ X_6 \\ Y_6 \end{matrix}$$

$$\begin{array}{c|cc|cc|cc|cc|cc|cc} & 1 & & 2 & & 3 & & 4 & & 5 & & 6 & \\ \hline 1 & 8.78E+7 & 1.75E+7 & -2.44E+7 & 1.54E+7 & 0.00E+0 & 0.00E+0 & -3.97E+7 & -9.29E+6 & -2.36E+7 & -2.36E+7 & 0.00E+0 & 0.00E+0 \\ & 1.75E+7 & 1.46E+8 & 2.20E+7 & -5.78E+7 & 0.00E+0 & 0.00E+0 & -1.60E+7 & -1.12E+8 & -2.36E+7 & 2.36E+7 & 0.00E+0 & 0.00E+0 \\ \hline 2 & -2.44E+7 & 2.20E+7 & 2.34E+8 & -1.21E+7 & -5.78E+7 & 1.54E+7 & 0.00E+0 & 0.00E+0 & -1.52E+8 & -2.52E+7 & 0.00E+0 & 0.00E+0 \\ & 1.54E+7 & -5.78E+7 & -1.21E+7 & 2.34E+8 & 2.20E+7 & -2.44E+7 & 0.00E+0 & 0.00E+0 & -2.52E+7 & -1.52E+8 & 0.00E+0 & 0.00E+0 \\ \hline 3 & 0.00E+0 & 0.00E+0 & -5.78E+7 & 2.20E+7 & 1.46E+8 & 1.75E+7 & 0.00E+0 & 0.00E+0 & 2.36E+7 & -2.36E+7 & -1.12E+8 & -1.60E+7 \\ & 0.00E+0 & 0.00E+0 & 1.54E+7 & -2.44E+7 & 1.75E+7 & 8.78E+7 & 0.00E+0 & 0.00E+0 & -2.36E+7 & -2.36E+7 & -9.29E+6 & -3.97E+7 \\ \hline 4 & -3.97E+7 & -1.60E+7 & 0.00E+0 & 0.00E+0 & 0.00E+0 & 0.00E+0 & 4.28E+7 & -4.04E+6 & -3.05E+6 & 2.00E+7 & 0.00E+0 & 0.00E+0 \\ & -9.29E+6 & -1.12E+8 & 0.00E+0 & 0.00E+0 & 0.00E+0 & 0.00E+0 & -4.04E+6 & 1.13E+8 & 1.33E+7 & -1.14E+6 & 0.00E+0 & 0.00E+0 \\ \hline 5 & -2.36E+7 & -2.36E+7 & -1.52E+8 & -2.52E+7 & 2.36E+7 & -2.36E+7 & -3.05E+6 & 1.33E+7 & 1.56E+8 & 3.91E+7 & -1.14E+6 & 2.00E+7 \\ & -2.36E+7 & 2.36E+7 & -2.52E+7 & -1.52E+8 & -2.36E+7 & -2.36E+7 & 2.00E+7 & -1.14E+6 & 3.91E+7 & 1.56E+8 & 1.33E+7 & -3.05E+6 \\ \hline 6 & 0.00E+0 & 0.00E+0 & 0.00E+0 & 0.00E+0 & -1.12E+8 & -9.29E+6 & 0.00E+0 & 0.00E+0 & -1.14E+6 & 1.33E+7 & 1.13E+8 & -4.04E+6 \\ & 0.00E+0 & 0.00E+0 & 0.00E+0 & 0.00E+0 & -1.60E+7 & -3.97E+7 & 0.00E+0 & 0.00E+0 & 2.00E+7 & -3.05E+6 & -4.04E+6 & 4.28E+7 \end{array} \quad \begin{array}{c} 0 \\ v_1 \\ \hline u_2 \\ v_2 \\ \hline u_3 \\ 0 \\ \hline 0 \\ v_4 \\ \hline u_5 \\ v_5 \\ \hline u_6 \\ 0 \end{array} = \begin{array}{c} 2929 \\ 7071 \\ \hline 10000 \\ 10000 \\ \hline 7071 \\ 2929 \\ \hline 0 \\ 0 \\ \hline 0 \\ 0 \\ \hline 0 \\ 0 \end{array}$$

Global Stiffness Matrix:

Accuracy Check for Element and Global Stiffness Matrix:

1. It should be symmetric
2. Each of the diagonal coefficient should be > 0
3. Sum of each row and column should be zero

Global stiffness matrix is always symmetric and positive definite for structural problems and for governing differential equation that is self-adjoint. The diagonal coefficients are always positive and relatively large when compared to the off-diagonal values in the same row.

Fixity Imposition through "Prescribed Displacement Conditions"

Method-1:

$$[K]_{N,N} \cdot \{\delta\}_{N,1} = \{P\}_{N,1}$$

Row vector corresponding to presribed variable is deleted while column vector is multiplied by α (the prescribed displacement value) & transferred to RHS.

$$[K]_{N-1,N-1} \cdot \{\delta\}_{N-1,1} = \{P\}_{N-1,1} - \alpha \cdot \{k_{1n}\}_{N-1,1}$$

$$\begin{pmatrix} k_{11} & k_{12} & \dots & k_{1n} & \dots & k_{1N} \\ k_{21} & k_{22} & \dots & k_{2n} & \dots & k_{2N} \\ . & . & \dots & . & \dots & . \\ k_{n1} & k_{n2} & \dots & k_{nn} & \dots & k_{nN} \\ . & . & \dots & . & \dots & . \\ k_{N1} & k_{N1} & \dots & k_{Nn} & \dots & k_{NN} \end{pmatrix} \begin{Bmatrix} \delta 1 \\ \delta 2 \\ . \\ \delta n \\ \\ \delta N \end{Bmatrix} = \begin{Bmatrix} P1 \\ P2 \\ . \\ Pn \\ . \\ PN \end{Bmatrix}$$

$$\Downarrow$$

$$\begin{pmatrix} k_{11} & k_{12} & \dots & k_{1n} & \dots & k_{1N} \\ k_{21} & k_{22} & \dots & k_{2n} & \dots & k_{2N} \\ . & . & \dots & . & \dots & . \\ \cancel{kn1} & \cancel{kn2} & \cancel{\dots} & \cancel{knn} & \cancel{\dots} & \cancel{knN} \\ . & . & \dots & . & \dots & . \\ k_{N1} & k_{N1} & \dots & k_{Nn} & \dots & k_{NN} \end{pmatrix} \begin{Bmatrix} \delta 1 \\ \delta 2 \\ . \\ \delta n \\ \\ \delta N \end{Bmatrix} = \begin{Bmatrix} P1 \\ P2 \\ . \\ \cancel{Pn} \\ . \\ PN \end{Bmatrix} - \alpha \cdot \begin{Bmatrix} k_{1n} \\ k_{2n} \\ . \\ \cancel{knn} \\ . \\ kNN \end{Bmatrix}$$

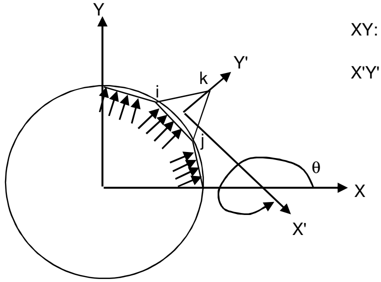
Reaction corresponding to prescribed displacement δn

$$R_n = \sum_{i=1,N} k_{ni}\, \delta_i - P_n$$

$$\Downarrow$$

In case α = 0, both rows & columns are deleted

Load Vector and Boundary Conditions:



XY: Global Co-ordinate System

X'Y': Local Co-ordinate System

Load Summary

Nodal Load

Node No.	F_X	F_Y
1	2929	7071
2	10000	10000
3	7071	2929

Boundary Element	Nodes on Boundary		Corordinates				Edge Length	COS(θ)	SIN(θ)	Pressure	F_X	F_Y
	i	j	X_i	Y_i	X_j	Y_j	L	---	---	q	Equal for each node	
1	**1**	**2**	0	0.2	0.14142	0.14142	0.1531	0.9239	-0.3827	100000	**2929**	**7071**
3	**2**	**3**	0.14142	0.14142	0.2	0	0.1531	0.3827	-0.9239	100000	**7071**	**2929**

Boundary Conditions:

Due to symmetry, x-displacement u, on face 1-4 is zero.
Due to symmetry, y-displacement v, on face 3-6 is zero.

u1 = 0
u4 = 0
v3 = 0
v6 = 0

Gaussian Elimination:

Gaussian Elimination:

$$
\begin{array}{c|cccccccc|}
 & 1v & 2u & 2v & 3u & 4v & 5u & 5v & 6u \\
1v & \mathbf{1.46E+08} & 2.20E+07 & -5.78E+07 & 0.00E+00 & -1.12E+08 & -2.36E+07 & 2.36E+07 & 0.00E+00 \\
2u & 2.20E+07 & 2.34E+08 & -1.21E+07 & -5.78E+07 & 0.00E+00 & -1.52E+08 & -2.52E+07 & 0.00E+00 \\
2v & -5.78E+07 & -1.21E+07 & 2.34E+08 & 2.20E+07 & 0.00E+00 & -2.52E+07 & -1.52E+08 & 0.00E+00 \\
3u & 0.00E+00 & -5.78E+07 & 2.20E+07 & 1.46E+08 & 0.00E+00 & 2.36E+07 & -2.36E+07 & -1.12E+08 \\
4v & -1.12E+08 & 0.00E+00 & 0.00E+00 & 0.00E+00 & 1.13E+08 & 1.33E+07 & -1.14E+06 & 0.00E+00 \\
5u & -2.36E+07 & -1.52E+08 & -2.52E+07 & 2.36E+07 & 1.33E+07 & 1.56E+08 & 3.91E+07 & -1.14E+06 \\
5v & 2.36E+07 & -2.52E+07 & -1.52E+08 & -2.36E+07 & -1.14E+06 & 3.91E+07 & 1.56E+08 & 1.33E+07 \\
6u & 0.00E+00 & 0.00E+00 & 0.00E+00 & -1.12E+08 & 0.00E+00 & -1.14E+06 & 1.33E+07 & 1.13E+08
\end{array}
\begin{Bmatrix} v1 \\ u2 \\ v2 \\ u3 \\ v4 \\ u5 \\ v5 \\ u6 \end{Bmatrix}
=
\begin{Bmatrix} 7.07E+03 \\ 1.00E+04 \\ 1.00E+04 \\ 7.07E+03 \\ 0.00E+00 \\ 0.00E+00 \\ 0.00E+00 \\ 0.00E+00 \end{Bmatrix}
$$

Step-1:

$$
\begin{array}{c|cccccccc|}
 & 1v & 2u & 2v & 3u & 4v & 5u & 5v & 6u \\
1v & \mathbf{1.46E+08} & 2.20E+07 & -5.78E+07 & 0.00E+00 & -1.12E+08 & -2.36E+07 & 2.36E+07 & 0.00E+00 \\
2u & 0 & \mathbf{2.31E+08} & -3.44E+06 & -5.78E+07 & 1.69E+07 & -1.48E+08 & -2.88E+07 & 0.00E+00 \\
2v & 0 & -3.44E+06 & 2.11E+08 & 2.20E+07 & -4.43E+07 & -3.46E+07 & -1.43E+08 & 0.00E+00 \\
3u & 0 & -5.78E+07 & 2.20E+07 & 1.46E+08 & 0.00E+00 & 2.36E+07 & -2.36E+07 & -1.12E+08 \\
4v & 0 & 1.69E+07 & -4.43E+07 & 0.00E+00 & 2.74E+07 & -4.73E+06 & 1.69E+07 & 0.00E+00 \\
5u & 0 & -1.48E+08 & -3.46E+07 & 2.36E+07 & -4.73E+06 & 1.52E+08 & 4.28E+07 & -1.14E+06 \\
5v & 0 & -2.88E+07 & -1.43E+08 & -2.36E+07 & 1.69E+07 & 4.28E+07 & 1.52E+08 & 1.33E+07 \\
6u & 0 & 0.00E+00 & 0.00E+00 & -1.12E+08 & 0.00E+00 & -1.14E+06 & 1.33E+07 & 1.13E+08
\end{array}
\begin{Bmatrix} v1 \\ u2 \\ v2 \\ u3 \\ v4 \\ u5 \\ v5 \\ u6 \end{Bmatrix}
=
\begin{Bmatrix} 7.07E+03 \\ 8.94E+03 \\ 1.28E+04 \\ 7.07E+03 \\ 5.42E+03 \\ 1.14E+03 \\ -1.14E+03 \\ 0.00E+00 \end{Bmatrix}
$$

Step-2:

$$
\begin{array}{c|cccccccc|}
 & 1v & 2u & 2v & 3u & 4v & 5u & 5v & 6u \\
1v & 1.46E+08 & 2.20E+07 & -5.78E+07 & 0.00E+00 & -1.12E+08 & -2.36E+07 & 2.36E+07 & 0.00E+00 \\
2u & 0 & \mathbf{2.31E+08} & -3.44E+06 & -5.78E+07 & 1.69E+07 & -1.48E+08 & -2.88E+07 & 0.00E+00 \\
2v & 0 & 0 & \mathbf{2.11E+08} & 2.12E+07 & -4.40E+07 & -3.68E+07 & -1.43E+08 & 0.00E+00 \\
3u & 0 & 0 & 2.12E+07 & 1.32E+08 & 4.22E+06 & -1.36E+07 & -3.08E+07 & -1.12E+08 \\
4v & 0 & 0 & -4.40E+07 & 4.22E+06 & 2.61E+07 & 6.12E+06 & 1.90E+07 & 0.00E+00 \\
5u & 0 & 0 & -3.68E+07 & -1.36E+07 & 6.12E+06 & 5.69E+07 & 2.43E+07 & -1.14E+06 \\
5v & 0 & 0 & -1.43E+08 & -3.08E+07 & 1.90E+07 & 2.43E+07 & 1.49E+08 & 1.33E+07 \\
6u & 0 & 0 & 0.00E+00 & -1.12E+08 & 0.00E+00 & -1.14E+06 & 1.33E+07 & 1.13E+08
\end{array}
\begin{Bmatrix} v1 \\ u2 \\ v2 \\ u3 \\ v4 \\ u5 \\ v5 \\ u6 \end{Bmatrix}
=
\begin{Bmatrix} 7.07E+03 \\ 8.94E+03 \\ 1.29E+04 \\ 9.31E+03 \\ 4.76E+03 \\ 6.88E+03 \\ -2.40E+01 \\ 0.00E+00 \end{Bmatrix}
$$

Step-3:

$$
\begin{array}{c|cccccccc|}
 & 1v & 2u & 2v & 3u & 4v & 5u & 5v & 6u \\
1v & 1.46E+08 & 2.20E+07 & -5.78E+07 & 0.00E+00 & -1.12E+08 & -2.36E+07 & 2.36E+07 & 0.00E+00 \\
2u & 0 & 2.31E+08 & -3.44E+06 & -5.78E+07 & 1.69E+07 & -1.48E+08 & -2.88E+07 & 0.00E+00 \\
2v & 0 & 0 & \mathbf{2.11E+08} & 2.12E+07 & -4.40E+07 & -3.68E+07 & -1.43E+08 & 0.00E+00 \\
3u & 0 & 0 & 0 & \mathbf{1.30E+08} & 8.63E+06 & -9.88E+06 & -1.65E+07 & -1.12E+08 \\
4v & 0 & 0 & 0 & 8.63E+06 & 1.69E+07 & -1.54E+06 & -1.08E+07 & 0.00E+00 \\
5u & 0 & 0 & 0 & -9.88E+06 & -1.54E+06 & 5.05E+07 & -5.41E+05 & -1.14E+06 \\
5v & 0 & 0 & 0 & -1.65E+07 & -1.08E+07 & -5.41E+05 & 5.19E+07 & 1.33E+07 \\
6u & 0 & 0 & 0 & -1.12E+08 & 0.00E+00 & -1.14E+06 & 1.33E+07 & 1.13E+08
\end{array}
\begin{Bmatrix} v1 \\ u2 \\ v2 \\ u3 \\ v4 \\ u5 \\ v5 \\ u6 \end{Bmatrix}
=
\begin{Bmatrix} 7.07E+03 \\ 8.94E+03 \\ 1.29E+04 \\ 8.01E+03 \\ 7.46E+03 \\ 9.13E+03 \\ 8.73E+03 \\ 0.00E+00 \end{Bmatrix}
$$

Step-4:

$$
\begin{array}{c|cccccccc|}
 & 1v & 2u & 2v & 3u & 4v & 5u & 5v & 6u \\
1v & 1.46E+08 & 2.20E+07 & -5.78E+07 & 0.00E+00 & -1.12E+08 & -2.36E+07 & 2.36E+07 & 0.00E+00 \\
2u & 0 & 2.31E+08 & -3.44E+06 & -5.78E+07 & 1.69E+07 & -1.48E+08 & -2.88E+07 & 0.00E+00 \\
2v & 0 & 0 & 2.11E+08 & 2.12E+07 & -4.40E+07 & -3.68E+07 & -1.43E+08 & 0.00E+00 \\
3u & 0 & 0 & 0 & \mathbf{1.30E+08} & 8.63E+06 & -9.88E+06 & -1.65E+07 & -1.12E+08 \\
4v & 0 & 0 & 0 & 0 & \mathbf{1.64E+07} & -8.85E+05 & -9.68E+06 & 7.46E+06 \\
5u & 0 & 0 & 0 & 0 & -8.85E+05 & 4.98E+07 & -1.79E+06 & -9.68E+06 \\
5v & 0 & 0 & 0 & 0 & -9.68E+06 & -1.79E+06 & 4.98E+07 & -8.85E+05 \\
6u & 0 & 0 & 0 & 0 & 7.46E+06 & -9.68E+06 & -8.85E+05 & 1.64E+07
\end{array}
\begin{Bmatrix} v1 \\ u2 \\ v2 \\ u3 \\ v4 \\ u5 \\ v5 \\ u6 \end{Bmatrix}
=
\begin{Bmatrix} 7.07E+03 \\ 8.94E+03 \\ 1.29E+04 \\ 8.01E+03 \\ 6.92E+03 \\ 9.74E+03 \\ 9.74E+03 \\ 6.92E+03 \end{Bmatrix}
$$

Step-5:

$$
\begin{array}{c|cccccccc|}
 & 1v & 2u & 2v & 3u & 4v & 5u & 5v & 6u \\
1v & 1.46E+08 & 2.20E+07 & -5.78E+07 & 0.00E+00 & -1.12E+08 & -2.36E+07 & 2.36E+07 & 0.00E+00 \\
2u & 0 & 2.31E+08 & -3.44E+06 & -5.78E+07 & 1.69E+07 & -1.48E+08 & -2.88E+07 & 0.00E+00 \\
2v & 0 & 0 & 2.11E+08 & 2.12E+07 & -4.40E+07 & -3.68E+07 & -1.43E+08 & 0.00E+00 \\
3u & 0 & 0 & 0 & 1.30E+08 & 8.63E+06 & -9.88E+06 & -1.65E+07 & -1.12E+08 \\
4v & 0 & 0 & 0 & 0 & \mathbf{1.64E+07} & -8.85E+05 & -9.68E+06 & 7.46E+06 \\
5u & 0 & 0 & 0 & 0 & 0 & \mathbf{4.97E+07} & -2.32E+06 & -9.28E+06 \\
5v & 0 & 0 & 0 & 0 & 0 & -2.32E+06 & 4.41E+07 & 3.53E+06 \\
6u & 0 & 0 & 0 & 0 & 0 & -9.28E+06 & 3.53E+06 & 1.30E+07
\end{array}
\begin{Bmatrix} v1 \\ u2 \\ v2 \\ u3 \\ v4 \\ u5 \\ v5 \\ u6 \end{Bmatrix}
=
\begin{Bmatrix} 7.07E+03 \\ 8.94E+03 \\ 1.29E+04 \\ 8.01E+03 \\ 6.92E+03 \\ 1.01E+04 \\ 1.38E+04 \\ 3.77E+03 \end{Bmatrix}
$$

Step-6:

$$
\begin{array}{c|cccccccc|}
 & 1v & 2u & 2v & 3u & 4v & 5u & 5v & 6u \\
1v & 1.46E+08 & 2.20E+07 & -5.78E+07 & 0.00E+00 & -1.12E+08 & -2.36E+07 & 2.36E+07 & 0.00E+00 \\
2u & & 2.31E+08 & -3.44E+06 & -5.78E+07 & 1.69E+07 & -1.48E+08 & -2.88E+07 & 0.00E+00 \\
2v & 0 & 0 & 2.11E+08 & 2.12E+07 & -4.40E+07 & -3.68E+07 & -1.43E+08 & 0.00E+00 \\
3u & 0 & 0 & 0 & 1.30E+08 & 8.63E+06 & -9.88E+06 & -1.65E+07 & -1.12E+08 \\
4v & 0 & 0 & 0 & 0 & 1.64E+07 & -8.85E+05 & -9.68E+06 & 7.46E+06 \\
5u & 0 & 0 & 0 & 0 & 0 & \mathbf{4.97E+07} & -2.32E+06 & -9.28E+06 \\
5v & 0 & 0 & 0 & 0 & 0 & 0 & \mathbf{4.40E+07} & 3.10E+06 \\
6u & 0 & 0 & 0 & 0 & 0 & 0 & 3.10E+06 & 1.12E+07 \\
 & 1v & 2u & 2v & 3u & 4v & 5u & 5v & 6u
\end{array}
\begin{Bmatrix} v1 \\ u2 \\ v2 \\ u3 \\ v4 \\ u5 \\ v5 \\ u6 \end{Bmatrix}
=
\begin{Bmatrix} 7.07E+03 \\ 8.94E+03 \\ 1.29E+04 \\ 8.01E+03 \\ 6.92E+03 \\ 1.01E+04 \\ 1.43E+04 \\ 5.66E+03 \end{Bmatrix}
$$

Gaussian Elimination:

Step-7:

$$\begin{array}{c|cccccccc|}
1v & 1.46E+08 & 2.20E+07 & -5.78E+07 & 0.00E+00 & -1.12E+08 & -2.36E+07 & 2.36E+07 & 0.00E+00 \\
2u & & 2.31E+08 & -3.44E+06 & -5.78E+07 & 1.69E+07 & -1.48E+08 & -2.88E+07 & 0.00E+00 \\
2v & 0 & 0 & 2.11E+08 & 2.12E+07 & -4.40E+07 & -3.68E+07 & -1.43E+08 & 0.00E+00 \\
3u & 0 & 0 & 0 & 1.30E+08 & 8.63E+06 & -9.88E+06 & -1.65E+07 & -1.12E+08 \\
4v & 0 & 0 & 0 & 0 & 1.64E+07 & -8.85E+05 & -9.68E+06 & 7.46E+06 \\
5u & 0 & 0 & 0 & 0 & 0 & 4.97E+07 & -2.32E+06 & -9.28E+06 \\
5v & 0 & 0 & 0 & 0 & 0 & 0 & \mathbf{4.40E+07} & 3.10E+06 \\
6u & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1.10E+07
\end{array}
\left\{\begin{array}{c} v1 \\ u2 \\ v2 \\ u3 \\ v4 \\ u5 \\ v5 \\ u6 \end{array}\right\}
=
\left\{\begin{array}{c} 7.07E+03 \\ 8.94E+03 \\ 1.29E+04 \\ 8.01E+03 \\ 6.92E+03 \\ 1.01E+04 \\ 1.48E+04 \\ 4.65E+03 \end{array}\right\}$$

$$\left\{\begin{array}{c} u1 \\ v1 \\ u2 \\ v2 \\ u3 \\ v3 \\ u4 \\ v4 \\ u5 \\ v5 \\ u6 \\ v6 \end{array}\right\}
=
\left\{\begin{array}{c} 0.00E+00 \\ 4.65E-04 \\ 3.56E-04 \\ 3.63E-04 \\ 4.59E-04 \\ 0.00E+00 \\ 0.00E+00 \\ 4.28E-04 \\ 2.96E-04 \\ 3.07E-04 \\ 4.22E-04 \\ 0.00E+00 \end{array}\right\}
\begin{array}{c} 0.00000 \\ 0.46453 \\ 0.35646 \\ 0.36347 \\ 0.45912 \\ 0.00000 \\ 0.00000 \\ 0.42806 \\ 0.29631 \\ 0.30654 \\ 0.42169 \\ 0.00000 \end{array}$$

m mm

Gaussian Elimination:

↓

Elemental Stress Calculation:

[D] . [B] $\{\delta\}$ = $\{\sigma\}$

↓

Stress Values → Graphical representation (Post-processing)

Elemental Stress Calculation: [D] . [B] . $\{\delta\}$ = $\{\sigma\}$

Element-1:

[D] . [B]

$$\begin{array}{cccccc} 1 & & 2 & & 5 & \end{array}$$

$$\begin{pmatrix}
-5.33E+11 & 1.33E+11 & 9.15E+10 & -4.00E+11 & 4.42E+11 & 2.67E+11 \\
-1.33E+11 & 5.33E+11 & 2.29E+10 & -1.60E+12 & 1.10E+11 & 1.07E+12 \\
2.00E+11 & -2.00E+11 & -6.00E+11 & 3.43E+10 & 4.00E+11 & 1.66E+11
\end{pmatrix}
\quad
\{\delta\}\left\{\begin{array}{c} 0.00E+00 \\ 4.65E-04 \\ 3.56E-04 \\ 3.63E-04 \\ 2.96E-04 \\ 3.07E-04 \end{array}\right\}
\begin{array}{c} 1 \\ \\ 2 \\ \\ 5 \\ \\ \end{array}$$

% Error	$\{\sigma\}$	Theoretical
-13.1	1.62E+08	1.43E+08
-102.0	3.40E+07	1.69E+07
8.3	-1.25E+08	-1.36E+08

Element-2:

$$\begin{array}{cccccc} 1 & & 5 & & 4 & \end{array}$$

$$\begin{pmatrix}
-4.42E+11 & -2.67E+11 & 5.03E+11 & 0.00E+00 & -6.10E+10 & 2.67E+11 \\
-1.10E+11 & -1.07E+12 & 1.26E+11 & 0.00E+00 & -1.52E+10 & 1.07E+12 \\
-4.00E+11 & -1.66E+11 & 0.00E+00 & 1.89E+11 & 4.00E+11 & -2.29E+10
\end{pmatrix}
\quad
\{\delta\}\left\{\begin{array}{c} 0.00E+00 \\ 4.65E-04 \\ 2.96E-04 \\ 3.07E-04 \\ 0.00E+00 \\ 4.28E-04 \end{array}\right\}
\begin{array}{c} 1 \\ \\ 5 \\ \\ 4 \\ \\ \end{array}
\quad
\{\sigma\}\left\{\begin{array}{c} 1.39E+08 \\ -1.66E+06 \\ -2.90E+07 \end{array}\right\}$$

Element-3:

$$\begin{array}{cccccc} 2 & & 3 & & 5 & \end{array}$$

$$\begin{pmatrix}
-1.60E+12 & 2.29E+10 & 5.33E+11 & -1.33E+11 & 1.07E+12 & 1.10E+11 \\
-4.00E+11 & 9.15E+10 & 1.33E+11 & -5.33E+11 & 2.67E+11 & 4.42E+11 \\
3.43E+10 & -6.00E+11 & -2.00E+11 & 2.00E+11 & 1.66E+11 & 4.00E+11
\end{pmatrix}
\quad
\{\delta\}\left\{\begin{array}{c} 3.56E-04 \\ 3.63E-04 \\ 4.59E-04 \\ 0.00E+00 \\ 2.96E-04 \\ 3.07E-04 \end{array}\right\}
\begin{array}{c} 2 \\ \\ 3 \\ \\ 5 \\ \\ \end{array}
\quad
\{\sigma\}\left\{\begin{array}{c} 3.28E+07 \\ 1.66E+08 \\ -1.26E+08 \end{array}\right\}$$

Element-4:

$$\begin{array}{cccccc} 3 & & 6 & & 5 & \end{array}$$

$$\begin{pmatrix}
-1.07E+12 & -1.10E+11 & 1.07E+12 & -1.52E+10 & 0.00E+00 & 1.26E+11 \\
-2.67E+11 & -4.42E+11 & 2.67E+11 & -6.10E+10 & 0.00E+00 & 5.03E+11 \\
-1.66E+11 & -4.00E+11 & -2.29E+10 & 4.00E+11 & 1.89E+11 & 0.00E+00
\end{pmatrix}
\quad
\{\delta\}\left\{\begin{array}{c} 4.59E-04 \\ 0.00E+00 \\ 4.22E-04 \\ 0.00E+00 \\ 2.96E-04 \\ 3.07E-04 \end{array}\right\}
\begin{array}{c} 3 \\ \\ 6 \\ \\ 5 \\ \\ \end{array}
\quad
\{\sigma\}\left\{\begin{array}{c} -1.39E+06 \\ 1.44E+08 \\ -2.98E+07 \end{array}\right\}$$

Method-2:

Diagonal co-efficient corresponding to δ_n is made unity & rest of the coefficients in the row area set = 0. To maintain symmetry in the matrix, corresponding column in multiplied by α and transferred to RHS as described in method-1. After this the column is also set = 0 except the diagonal element. Lastly the coefficient P_n is replaced by α and the final form of equations is shown below:

$$\begin{array}{l} \\ \\ \\ \textbf{nth row} \\ \\ \\ \end{array}\begin{pmatrix} k_{11} & k_{12} & \ldots & 0 & \ldots & k_{1N} \\ k_{21} & k_{22} & \ldots & 0 & \ldots & k_{2N} \\ . & . & \ldots & . & \ldots & . \\ 0 & 0 & \ldots & 1 & \ldots & 0 \\ . & . & \ldots & . & \ldots & . \\ k_{N1} & k_{N1} & \ldots & 0 & \ldots & k_{NN} \end{pmatrix} \begin{Bmatrix} \delta 1 \\ \delta 2 \\ . \\ \delta n \\ . \\ \delta N \end{Bmatrix} = \begin{Bmatrix} P_1 \\ P_2 \\ . \\ \alpha \\ . \\ P_N \end{Bmatrix} - \alpha . \begin{Bmatrix} k_{1n} \\ k_{2n} \\ . \\ 0 \\ . \\ k_{Nn} \end{Bmatrix}$$

Reaction corresponding to prescribed displacement δn

$$R_n = \quad \sum_{i=1,\,N} k_{ni}\, \delta_i - P_n$$

Method-3:

This method requires very few operations and is preferable to method 4 for non-zero displacements and when reactions are not required. In this method, the diagonal coefficients corresponding to $\boldsymbol{\delta_n}$ is multiplied by a very large number, say 10^{25}, and the load term Pn is replaced by α x k_{nn} x 10^{25}.

$$\begin{array}{l} \\ \\ \\ \text{nth row} \\ \\ \\ \end{array}\begin{pmatrix} k_{11} & k_{12} & \ldots & k_{1n} & \ldots & k_{1N} \\ k_{21} & k_{22} & \ldots & k_{1n} & \ldots & k_{2N} \\ . & . & \ldots & . & \ldots & . \\ k_{n1} & k_{n2} & \ldots & k_{nn} \text{ x } 10^{25} & \ldots & k_{nN} \\ . & . & \ldots & . & \ldots & . \\ k_{N1} & k_{N2} & \ldots & k_{Nn} & \ldots & k_{NN} \end{pmatrix} \begin{Bmatrix} \delta_1 \\ \delta_2 \\ . \\ \delta_n \\ . \\ \delta_N \end{Bmatrix} = \begin{Bmatrix} P_1 \\ P_2 \\ . \\ \alpha \text{ x } k_{nn} \text{ x } 10^{25} \\ . \\ P_N \end{Bmatrix}$$

Allowing for the fact that the coefficients in a row are approximately of the same order of magnitude, it is quite simple to see that the equation corresponding to δ_n is in fact very nearly equivalent to $\delta_n = \alpha$. The reactions are obtained as before by storing the original equation and performin ghe calculations as given by equation:

$$R_n = \quad \sum_{i=1,\,N} k_{ni}\, \delta_i - P_n$$

Method-4:

This method consists of adding a very large number, say 10^{50}, to the diagonal coefficients which physically corresponds to 'earthing' the structure with a very stif spring. For a rigid support, one would obtain a very small displacement instead of an absolute zero 7 the reaction for that support can be computed directly as

Reaction = - (Big Spring Stiffness) x (Very Small Displacement)

For non-zero displacement, it is only necessary to modify the RHS as explained in method-2. The small displacement obtained in the solution must be replaced by the prescribed displacement before th displacement vector is used to calculate the element stresses. This method will failif the big spring stiffness is not significantly larger than the stiffness coefficients of the structure. However, with a value of 10^{50}, this is unlikely to occur in practice.

$$\begin{array}{l} \\ \\ \\ \text{nth row} \\ \\ \\ \end{array}\begin{pmatrix} k_{11}+10^{5} & k_{12} & \ldots & 0 & \ldots & k_{1N} \\ k_{21} & k_{22}+10^{50} & \ldots & 0 & \ldots & k_{2N} \\ . & . & \ldots & . & \ldots & . \\ k_{n1} & k_{n2} & \ldots & k_{nn}+10^{50} & \ldots & k_{nN} \\ . & . & \ldots & . & \ldots & . \\ k_{N1} & k_{N2} & \ldots & 0 & \ldots & k_{NN}+10^{50} \end{pmatrix} \begin{Bmatrix} \delta_1 \\ \delta_2 \\ . \\ \delta_n \\ . \\ \delta_N \end{Bmatrix} = \begin{Bmatrix} P_1 \\ P_2 \\ . \\ P_n \\ . \\ P_N \end{Bmatrix}$$